

# The Second Night







Robert was sliding. The same old story. It had started the minute his head hit the pillow, and he couldn't stop. This time he was sliding along a tree. Don't look down, he thought, clinging to the tree for dear life and sliding on and on . . .

Then all at once he landed with a plop on a soft bed of moss. He heard a giggle and who did he see but the number devil perched on a velvety brown mushroom, smaller than Robert had remembered, and staring down at him with his shining eyes.

"How did *you* get here?" he asked Robert.

Robert pointed to the tree trunk, which stretched as far as the eye could see. But it was not alone: there was a whole forest of them. And they weren't so much trees as ones. He had landed in a forest of ones.

But that was not all. The air hummed with tiny flylike numbers dancing in front of his nose. He tried shooping them away, but there were too many:

twos, threes, fours, fives, sixes, sevens, eights, and nines kept brushing against him. Robert had always hated moths and gnats and couldn't stand having the beasties around.

"Are they bothering you?" asked the number devil. He put his hand to his mouth and blew them all away with a *Pfft!* Suddenly the air was clear of everything but the forest of ones reaching up to the sky.

"Have a seat, Robert," said the number devil.

Robert was surprised to find him so friendly.

"Where? On a mushroom?"

"Why not?"

"But I'd feel silly. Where are we, anyway? In a picture book? Last time you sat on a spinach leaf, and now you're on a mushroom. I seem to remember something like that. In a book I once read."

"The mushroom in *Alice in Wonderland* perhaps."

"But what's the connection between a made-up story and mathematics?"

"The kind of connection you make when you dream, my boy. You don't think *I* was behind all those flies, do you? No, I'm wide awake. You're the one in bed dreaming. Now what do you say? Are you going to stand there forever?"



*Don't look down, Robert thought, clinging to the tree for dear life and sliding on and on . . . He had landed in a forest of ones.*

Robert saw he had to do something, so he clambered onto the nearest mushroom. It was enormous and, except for a few bumps, as soft and cozy as an armchair.

“How do you like it?”

“It’s fine,” said Robert. “I just wonder who came up with the number flies and the forest of ones. I couldn’t have. Not in my wildest dreams. It could only have been you.”

“And if it was?” said the number devil, preening himself on his mushroom. “Though there’s still something missing.”

“What?”

“Nothing. I mean, zero.”

He was right. There hadn’t been a single zero among all the flies.

“Why?”

“Because zero was the last number to be discovered. Which isn’t surprising, given that zero is the most sophisticated of numbers. Here, look.” And finding a space between two tree-high ones, he wrote some letters in the sky with his walking stick:

MCM

“Tell me, when were you born, Robert?”

“Me? In 1986,” said Robert a bit reluctantly.

1000 (100) 1000 50 10 10 10 5 1  
M C M L X X X V I

wrote the number devil.

“I know what those are. Those are those old numbers you sometimes find in the cemetery.”

“And they come from the ancient Romans. Who had a tough time of it, by the way. Partly because their numbers were so hard to read. Though this one is easy enough—”

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“One,” said Robert.

“And this?”

X

“X is ten.”

“Right. And this, my boy, is the year you were born:

M C M L X X X V I

The first M means 1,000. C is 100, but because it comes before the second M you must subtract it to

get 900. L is 50 and X is 10. You add them together, which gives you 80. V is 5, which you add to our friend 1 to get 6. So this is  $1,000 + 900 + 80 + 6$ .”

“Gosh, that’s awfully complicated.”

“Right again. And you know why? Because the Romans had no zero.”

“I don’t see the connection. Besides, what’s so great about zero? Zero means nothing.”

“Which is precisely what is so brilliant about it.”

“But why even call it a number? Nothing doesn’t count.”

“Don’t be so sure. Remember how we divided up that piece of chewing gum among all those billions of people and hundreds of billions of mice? And how the portions got so small that in the end you couldn’t see them, not even with a microscope? Well, we could have gone on forever without reaching zero. We’d have come closer and closer, but we’d never have made it.”

“So?”

“So we’ve got to try something else. Minus, for instance. Yes, that should do the trick.”

He stretched out his walking stick and tapped the end of one of the ones. It shrank and shrank until it stood meek and manageable at Robert’s feet.

*Idea of Limits*





“Go at it,” said the number devil.

“What do you mean?”

“Try the minus.”

$$1 - 1 =$$

“One minus one is zero,” said Robert. “Everyone knows that.”

“You see? You see how necessary zero is? You can’t do without it.”

“But why do we need to write it? If nothing is left, why not just leave a blank? Why invent a number for something that doesn’t exist?”

“Try this, then.”

$$1 - 2 =$$

“That’s easy,” Robert said. “One minus two is minus one.”

“Right. But look what you get without a zero.

$$\dots 4, 3, 2, 1, -1, -2, -3, -4 \dots$$

The difference between four and three is one. Between three and two—one. Between two and one—one. And between one and minus one?”

“Two.”

“Which means there must be a number missing between one and minus one.”

“That tricky little zero.”

“I told you we couldn’t do without it. That brings us back to the Romans. They reckoned they could, and look what happened. Instead of 1986, they had to fiddle with all those M’s, C’s, L’s, X’s, and V’s. The Romans had to give each number a different letter, all because they didn’t have zero.”

“But what’s that got to do with our chewing gum and the minus?”

“Forget the chewing gum. Forget the minus. The zero’s real beauty lies elsewhere. But you’ll need to use your head to appreciate it. Are you up for it or are you too tired?”

“No, as long as I’m not sliding I’m fine. In fact, I like it here on the mushroom.”

“Good. Then let me give you a little problem to solve.”

Why is he suddenly being so nice to me? Robert wondered. I bet he’s got something up his sleeve. But all he said was “Fire away.”

And the number devil asked:

$$9 + 1 =$$

“Ten,” Robert answered like a shot. “Is that all?”

“And how do you write it?”

“I haven’t got a pen.”

“Then skywrite it. Here, take my walking stick.”

$$9 + 1 = 10$$

Robert wrote in purple cloud script.

“One and zero?” the number devil said. “One plus zero doesn’t equal ten.”

“Oh, come off it!” Robert shouted. “I didn’t write one plus zero! I wrote a one and a zero, and that’s ten!”

“And why, may I ask, is that ten?”

“Because that’s the way you write it.”

“And why do you write it that way?”

“Why, why, why?” Robert moaned. “You’re getting on my nerves.”



“Want me to tell you?” the number devil asked, leaning back leisurely on his mushroom.

A long silence followed.

“Fine! Go ahead!” Robert blurted out when he couldn’t bear it any longer.

“Simple. It comes from hopping.”

“Hopping?” Robert said scornfully. “What’s that supposed to mean? Numbers don’t hop.”

“Numbers hop if *I* tell them to hop,” the number devil replied. “Don’t forget who you’re dealing with.”

“All right, all right,” Robert said. “Just tell me what you mean by hopping.”

“Gladly. Let’s go back to square one. Or, rather, the number one.

$$1 \times 1 = 1$$

$$1 \times 1 \times 1 = 1$$

$$1 \times 1 \times 1 \times 1 = 1$$

Tack on as many ones as you like and you still get one for your answer.”

“Sure. But what’s your point?”

“You’ll see if you try the same thing with two.”

“Okay,” said Robert.

$$2 \times 2 = 4$$

$$2 \times 2 \times 2 = 8$$

$$2 \times 2 \times 2 \times 2 = 16$$

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

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“Wow, that goes fast! If I go much further, I’ll need my calculator.”

“It goes even faster if you start with five. Why don’t you give it a try?”

$$5 \times 5 = 25$$

$$5 \times 5 \times 5 = 125$$

$$5 \times 5 \times 5 \times 5 = 625$$

$$5 \times 5 \times 5 \times 5 \times 5 = 3125$$

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15625$$

“Whoa!” Robert shouted.

“Why do large numbers make you so jumpy? I can assure you that most large numbers are perfectly harmless.”

“Says you!” said Robert. “Besides, I don’t see the point of multiplying five by itself over and over.”

“I’m coming to that. You know what the number devil does instead of writing all those boring fives? He writes:

$$\begin{aligned}5^1 &= 5 \\5^2 &= 25 \\5^3 &= 125\end{aligned}$$

and so on. Five to the first, five to the second, five to the third. In other words, I make the numbers hop.

“Now do you see? Do the same with ten, and it’s as easy as pie. You can throw your calculator away. Make the ten do one hop, and it remains exactly as is:

$$10^1 = 10$$

Make it hop twice, and you get:

$$10^2 = 100$$

Make it hop three times, and you get:

$$10^3 = 1000$$

“So if I make it hop five times,” Robert cried, “I get 100,000! Once more, and I get a million!”

“And so on, till the cows come home,” said the number devil. “Simple, eh? That’s the beauty of the zero. It lets you hold a space and move on. You can always tell a number’s value by its position: the farther to the left it is, the more it’s worth; the farther to the right, the less.

“When you write 555, you know the last five is worth exactly five and no more; the next-to-last five is worth ten times more—that is, fifty; and the first five is worth a hundred times the last one—that is, five hundred. And why? Because it’s been hopped up front.

“Now, the fives of the ancient Romans could never be anything but fives. Why? Because the Romans didn’t know how to hop. And why didn’t they know how to hop? Because they had no zero to keep places. Which meant they ended up with monstrosities like MCMLXXXVI.

“So rejoice, my boy! You’re much better off than the Romans. With the help of friend zero and

a bit of hopping you can produce any number, big or small, any number you please. 786, for instance.”

“But I don’t need 786.”

“Really now! You’re brighter than that! Try the year you were born, 1986.” The number devil started growing again, and his mushroom followed suit. “Well, what are you waiting for?” he bellowed. “Get a move on!”

There he goes again, thought Robert. Get him worked up about something and he’s impossible. Worse than Mr. Bockel.

He carefully wrote a large one in the sky.

“Wrong!” the number devil screamed. “Dead wrong! How did I wind up with a fool like you? I told you to *produce* the number, not scribble it down.”

Robert would have given anything to wake up. I’m not going to put up with this, he thought, watching the number devil’s head swell up and turn redder and redder.

“The end,” the number devil called down to him.

Robert stared back, completely at a loss.

“Start at the end, not the beginning.”

“If you say so,” said Robert, who was in no mood to argue. He erased the one and wrote a six in its place.





“Finally caught on, have you? Well then, we may proceed.”

“No problem,” said Robert warily, “though I’d appreciate it if you didn’t fly off the handle over every detail.”

“Sorry, but what do you expect? I’m the number devil, not Santa Claus.”

“How do you like my six?”

The number devil shook his head and wrote:

$$6 \times 1 = 6$$

“But that’s the same,” Robert protested.

“You’ll see what I have in mind. Now comes the eight. And don’t forget to hop.”

Suddenly Robert did see what he had in mind. He wrote:

$$8 \times 10 = 80$$

“I get it!” he shouted before the number devil could tell him what to do. “With nine I make the ten hop twice.” And he wrote:

$$9 \times 100 = 900$$

and

$$1 \times 1000 = 1000$$

“That was a triple hop,” he said. “And now for the grand total—

$$6 + 80 + 900 + 1000 = 1986$$

It’s not so hard after all. I don’t even need the number devil.”

“So you don’t need the number devil, eh? You’re getting too big for your breeches, boy! All you’ve had so far are the most ordinary numbers. Nothing to write home about. Wait till I start pulling numbers out of my hat, all kinds of numbers—unreal numbers, unreasonable numbers. You have no idea how many kinds of numbers there are! Numbers that run around in circles, numbers that play tricks with your brain, numbers that go on forever . . .”

As he spoke, his grin grew wider and wider. Robert could see all the teeth in his mouth; they too seemed to go on forever. And then he started twirling his walking stick in front of Robert’s face again.

“Help!” Robert screamed, and woke up.



*As the number devil spoke, his grin grew wider and wider. Robert could see all the teeth in his mouth; they too seemed to go on forever*

Still in a daze the next morning, Robert said to his mother, “Do you know the year I was born? It was  $6 \times 1$  and  $8 \times 10$  and  $9 \times 100$  and  $1 \times 1000$ .”

“I don’t know what’s got into the boy lately,” said Robert’s mother, shaking her head. “Here,” she added, handing him a cup of hot chocolate, “maybe this will help. You say the oddest things.”

Robert drank his hot chocolate in silence. There are some things you can’t tell your mother, he thought.

