

# The Third Night







In time Robert grew accustomed to dreaming of the number devil. He even came to look forward to it. True, he could have done without his know-it-all attitude and his temper tantrums—you could never tell when he'd blow up and yell at you—but it was better, so much better, than being swallowed by a slimy fish or sliding down and down into a black hole.

Besides, Robert had made up his mind to show the number devil that he was no fool. You have to put people like him in their place, Robert thought as he got ready for bed one night. The big ideas he has about himself—and all because of a zero. *He* wasn't much more than a zero when you got down to it. All you had to do was wake up and he was gone.

But to put him in his place Robert had to dream of him, and to dream of him he had to fall asleep. And

Robert suddenly noticed he was having trouble doing so. For the first time in his life he lay awake in bed, tossing and turning.

“What are you tossing and turning for?”

All at once, Robert realized his bed was in a cave. There were weird paintings of animals on the stone walls, but he had no time to study them because the number devil was standing over him, twirling his walking stick.

“Rise and shine, Robert!” he said. “Today’s our division day.”

“Must I?” Robert asked. “You might have at least waited until I was asleep. Besides, I hate division.”

“Why?”

“When you add or subtract or even multiply, things come out even. What bugs me about division is that you get this remainder.”

“The question is when.”

“When what?”

“When you get a remainder and when you don’t. That’s what counts. You can tell just by looking at them that some numbers can be divided evenly.”

“Right. Like even numbers, which can all be divided by two. No problem. I’m pretty good at threes as well:

$$9 \div 3$$
$$15 \div 3$$

and so on. It's like multiplying in reverse:

$$3 \times 5 = 15$$

becomes

$$15 \div 3 = 5$$

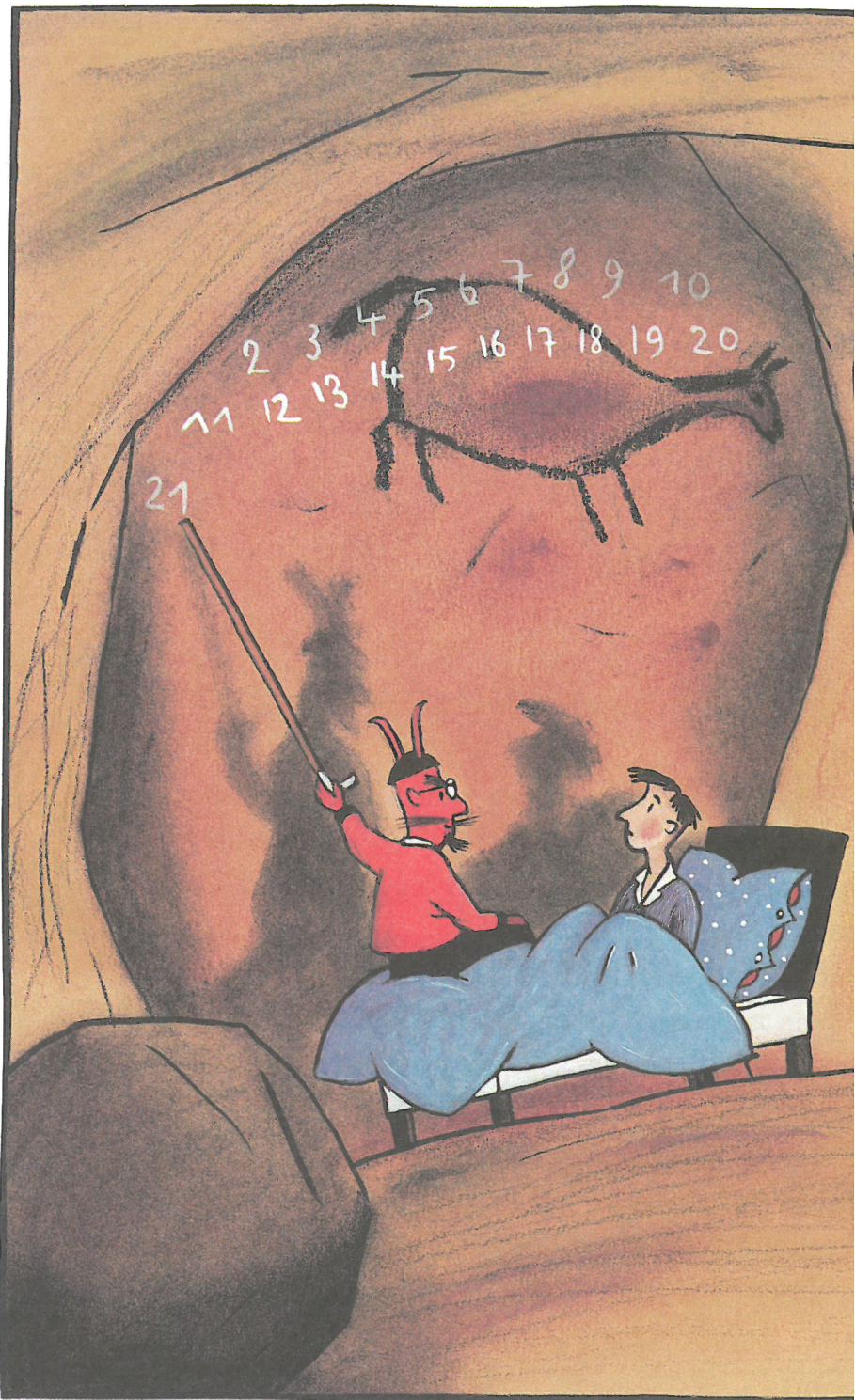
I don't need a number devil for that. I can do it on my own."

Robert shouldn't have said that. The number devil, his mustache quivering, his nose reddening, his head growing bigger and bigger, jerked Robert out of bed.

"What do *you* know?" the number devil shouted. "Just because you've learned the multiplication table you think you know all there is to know. Well, you know nothing! Nothing whatsoever!"

There he goes again, thought Robert. First he drags me out of bed, then he hits the ceiling when I tell him I can do division.





All things being equal, Robert would have upped and left, but how do you up a

“Here I come to a rank beginner out of the goodness of my heart, and no sooner do I open my mouth than he starts making wisecracks!”

“The goodness of your heart!” Robert cried. All things being equal, he would have upped and left, but how do you up and leave a dream? He looked all over the cave, but could find no way to leave.

“What are you looking for?”

“A way out.”

“If you go now, you’ll never see me again! I’ll leave you to choke on Mr. Bockel’s pretzel problems, or die of boredom in his class.”

Robert knew when he was licked.

“I apologize,” he said. “I didn’t mean to offend you.”

“Good,” said the number devil, his anger subsiding as quickly as it had come. “Now, nineteen. Try nineteen. See if you can divide it without a remainder.”

Robert thought and thought.

“The only way I can come up with,” he said at last, “is to divide it by nineteen. Or into nineteen equal parts.”

“Doesn’t count,” the number devil replied. “It’s too easy.”

“Or divide it by zero.”

“Out of the question.”

“Out of the question? Why?”

“Because it’s forbidden. Dividing by zero is strictly forbidden.”

“What if I did it anyway?”

“Then all mathematics would come apart at the seams!”

He was about to lose his temper again, but he managed to pull himself together.

“Tell me,” said the number devil, “what would you get if you divided nineteen by zero?”

“I don’t know. A hundred, maybe. Or zero. Or anything in between.”

“But didn’t you say when you were talking about the threes that division was like multiplying in reverse? If that’s the case, then

$$3 \times 5 = 15$$

means that

$$15 \div 3 = 5$$

Well, now try that with nineteen and zero.”

“Nineteen divided by zero is, say, 19.”

“And in reverse?”

“19 times zero . . . 19 times zero . . . is zero.”

“You see? And no matter what number you take, you always get zero. Which means you must never divide a number by zero.”



“Okay,” said Robert, “I give up. But what do we do with the nineteen? No matter what number I divide it by—two, three, four, five, six, seven, eight, or nine—I get stuck with a remainder.”

“Come a little closer,” said the number devil to Robert, “and I’ll tell you a secret.”

Robert leaned so close to the number devil that his mustache tickled his ear.

“There are two types of numbers,” he whispered. “The garden variety, which can be divided evenly,



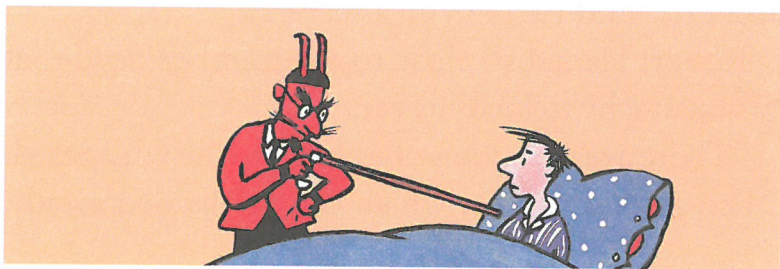
and the rest, which cannot. I much prefer the latter. You know why? Because they’re such prima donnas. From the very first they’ve caused mathematicians no end of trouble. Wonderful numbers those! Like eleven, thirteen, or seventeen.”

Robert couldn’t get over how blissful the number devil looked. He might have had a piece of chocolate melting in his mouth.

“Now tell me, dear boy, what the first prima-donna numbers are.”

“Zero,” said Robert, to get his dander up.

“Zero is forbidden!” the number devil shouted, brandishing his walking stick.



“All right, then. One.”

“One doesn’t count. How many times do I have to tell you?”

“Okay, okay,” said Robert. “Calm down! Two, to begin with. Because, like all prima donnas, it can only be divided by one and itself. And three—or at least I think so. Not four. We’ve been through that. Five for sure; five isn’t divisible by anything. And . . . and so on.”

“Ha! What is that supposed to mean?” he said, rubbing his hands together, a sure sign he had something up his sleeve. “The wonderful thing about prima donnas is that no one knows their ‘and so on.’ No one but me, of course. And I won’t tell.”

“Not even your friends?”

“Nobody! Never! The thing is, you can’t know whether a number is a prima donna or not merely by looking at it. No ordinary mortal can know without testing it.”

“And how do you test it?”

“You’ll see,” he said, and started scribbling all over the wall of the cave with his walking stick.

This is what it looked like when he was through:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

“Now take my stick, young man, and tap every number that isn’t a prima donna. That will make it disappear.”

“But there’s no zero,” Robert complained, “no one.”

“How many times must I tell you! Zero and one are unlike all other numbers: they are *neither*

prima donnas *nor* the ordinary kind. Don't you remember what you dreamed back at the very beginning? That all numbers come from one? And then later we saw the need for zero."

"Whatever you say," said Robert. "Anyway, first I'll tap the even numbers, because they're all divisible by two. That's easy."

"All the even numbers but two," the number devil warned him. "Don't forget: two is a prima-donna number."

Robert picked up the stick and in no time flat the wall looked like this:

	2	3	5	7	9	
11		13	15	17	19	
21		23	25	27	29	
31		33	35	37	39	
41		43	45	47	49	

"Now do three. Three is a prima-donna number, but the rest of the numbers in the three column of the multiplication table—six, nine, twelve, etcetera—are not, because they can be divided by three."

When Robert had taken care of three, the following numbers were left:

	2	3		5		7		
11		13				17		19
		23		25				29
31				35		37		
41		43				47		49

“Now four,” said Robert. “No, no. We don’t need to bother about numbers divisible by four: they’re all gone, because four isn’t a prima donna, four is  $2 \times 2$ . But five, five is a prima donna. Not ten, though, which is gone because it’s  $2 \times 5$ .”

“And you can tap all the ones that end in five.”

“Right.”

	2	3		5		7		
11		13				17		19
		23						29
31						37		
41		43				47		49

Robert was hitting his stride.

“We can forget about six—six is  $2 \times 3$ —but seven is a prima-donna number.”

“Good for you!” the number devil cried.

“Eleven too.”

“And what others?”

*That, my dear readers, is a question you yourselves must find the answer to. Copy the chart and go on tapping until only prima-donna numbers are left. Let me give you a hint: There are exactly fifteen of them in the chart—no more and no less.*

“Well done, Robert,” said the number devil, lighting his pipe and chuckling to himself.

“What’s so funny?” Robert asked.

“It’s not so hard if you stop at fifty,” he answered with a wicked grin and settling into a comfortable cross-legged position. “But what if you have a number like

10 000 019

or

141 421 356 237 307

Is it a prima donna or isn't it? If you knew how many mathematicians have racked their brains over the issue. Why, even the greatest number devils have come to grief over it."

"But I thought you said *you* knew the 'and so on' of prima-donna numbers. You just didn't want to let me in on it."

"Well, maybe I overstated my case a little."

"I'm glad you can admit you're not perfect," said Robert. "Sometimes you sound less like a number devil than a number dictator."

"The more simpleminded number devils use giant computers. They keep them running for months at a stretch. The trick I taught you—taking care of the twos and threes and fives first—is old hat really. Not that it doesn't work, but when the numbers start getting really big, you know that there's no end to them. We can make them grow bigger than the universe just by adding, multiplying, and hopping. Now there are all kinds of more sophisticated ways of doing things, but clever as they are they don't seem to get us very far. That's what makes them so devilishly interesting—and what is devilish is fun, don't you think?"

And so saying, he twirled his stick with great relish.

“Yes, but what’s the point of it all?” Robert asked.

“You *do* ask stupid questions! The world of numbers is never so musty as your Mr. Bockel—Mr. Pretzel—makes it out to be. Luckily you’ve got me to initiate you into some of the secrets. Like this, for instance: Take any number larger than one and multiply it by two.”

“222?” said Robert. “Times two is 444.”

“Between the first and the second number there is always—and when I say always, I mean *always*—at least one prima donna.”

“Are you sure?”

“307,” said the number devil. “But it works with gigantic numbers too.”

“Where do you learn these things?”

“You haven’t seen anything yet!” he said, savoring Robert’s curiosity. Nothing could stop him now. “Take any even number—any one at all, so long as it’s larger than two—and I can find two prima donnas that add up to it.”

“Forty-eight,” Robert said.

“Thirty-one plus seventeen,” said the number devil without blinking an eye.

“Thirty-four,” said Robert.



“Twenty-nine plus five,” said the number devil, without even taking the pipe out of his mouth.

“And it works all the time?” Robert asked, amazed. “Why? How come?”

“To tell you the truth,” the number devil said, his forehead wrinkling and his eyes looking upward at the smoke rings he was blowing, “I wish I knew. Nearly every number devil of my acquaintance has tried to come up with an explanation. It always works, but no one knows why.”

Pretty nifty, thought Robert. He laughed and said, “Well, I think it’s great.” But what he really thought was great was that the number devil had told him the inside story.

For a while the number devil had a rather crotchety expression on his face (as he always had when he didn’t know quite what to do), but when he heard Robert laugh he laughed too and started puffing on his pipe again.

“You’re not so stupid as you look, my boy. I’m sorry I have to leave you, but I’ve got a few more mathematicians to visit tonight. I get a kick out of torturing them a tad.”

With that, the number devil began to grow thin. No, not so much thin as transparent. And suddenly all that was left of him was a cloud of smoke



$$\begin{array}{r} 22 \\ \hline 11 + 11 \end{array}$$

$$\begin{array}{r} 55 \\ \hline 53 + 2 \end{array}$$

$$\begin{array}{r} 74 \\ \hline 37 + 37 \end{array}$$

and the scribbling on the wall. Then the wall started swimming before Robert's eyes and the cave felt as soft and warm as a blanket. Robert tried hard to remember what was so wonderful about prima-donna numbers, but his thoughts were all white and cloudy like a cotton mountain.

He had rarely slept so well.



*And you? Let me show you one last trick—if you haven't dozed off, that is. It works with odd as well as even numbers. Think of a number, any number, so long as it's bigger than five. Fifty-five, say, or twenty-seven.*

*You can find prima-donna numbers that add up to them too, only instead of two you'll need three. Let's use fifty-five as our example.*

$$55 = 5 + 19 + 31$$
$$27 = 5 + 11 + 11$$

*Now try twenty-seven. It always works—you'll see—though I can't explain why.*