The Fourth Night

"The places you drag me to! A cave with no opening, a forest with ones for trees and mushrooms the size of armchairs. What about today? Where am I anyway?"

"At the beach. Can't you tell?"

Robert looked around: white sand far and wide, the number devil perched on an overturned rowboat, the surf rolling in behind him, and not a soul in sight.

"I bet you've forgotten your calculator again," the number devil said.

"Look, how many times do I have to tell you? I can't take all my stuff to bed with me at night. Do *you* know what you're going to dream the night before you dream it?"

"Of course not," the number devil answered. "Still, if you dream of me, you can just as easily dream of your calculator. But no, I've got to come up with one by magic! I've got to do everything for you! And then you complain it's too soft or too green or too sticky!"

"It was better than nothing."

"All right. I'll try," the number devil said. He raised his wand, and a new calculator appeared before Robert's eyes. If the last one was like a frog, this one was like a soft, furry bed or sofa. And gigantic. At one end it had a little board with fur-covered number keys, but the screen for the numbers stretched the entire length of the back-rest. It was one weird calculator.

"Now enter one divided by three," the number devil ordered, "and see what you come up with."

$$1 \div 3$$

Robert punched the keys. The following answer came up in green on the long, long screen:

$$0,3333333333333333333$$

"Doesn't it ever stop?" Robert asked.

"Of course it does," the number devil answered. "It stops where the screen stops."

"And then what?"

"Then it goes on. You just can't see it."

"But it's always the same, one three after the next. How boring!"

"Right."

"And dumb too! It's much easier to write one-third:

$$\frac{1}{3}$$

Then I don't have to worry about all those threes creeping up."

"True," said the number devil, "but then you've got to deal with fractions, and fractions, if I'm not mistaken, are something you can't abide. 'If $\frac{1}{3}$ of 33 bakers can make 89 pretzels in $2\frac{1}{2}$ hours, then how many pretzels can $5\frac{3}{4}$ bakers make in $1\frac{1}{2}$ hours?' "

"No! No! Anything but that! Give me decimals any time! Even if the numbers never end. I'd just like to know what all those threes are doing there."

"Simple. The first three after the dot means three-tenths. The second means three-hundredths, the third three-thousandths, and so on. You can take it from there on your own.

$$
\begin{array}{l}
0,3 \\
0,03 \\
0,003 \\
0,0003 \\
0,00003 \\
\ldots
\end{array}
$$

Get it? Good. Then try multiplying everything by three: the three, the three-tenths, the three-hundredths, and so on."

"No problem," said Robert. "I can do that in my head."

$$
\begin{array}{r}
0,3 \times 3 = 0,9 \\
0,03 \times 3 = 0,09 \\
0,003 \times 3 = 0,009 \\
0,0003 \times 3 = 0,0009
\end{array}
$$

"Good. Now what happens if you add all the nines together?"

"Let's see: $0.9 + 0.09 = 0.99$, and $0.99 + 0.009 = 0.999$. Nines down the line. I bet it could keep on like that forever."

"Right you are. Though if you think about it, there's something fishy going on: ⅓ + ⅓ + ⅓ = 1, doesn't it? Because a third multiplied by three equals a whole. Always has and always will. Well? What do you think?"

"I don't know," said Robert. "Something is still missing—0.999 is *nearly* one, but it doesn't quite get there."

"That's the point. That's why you've got to keep the nines going and never stop."

"Easier said than done."

"Not for a number devil."

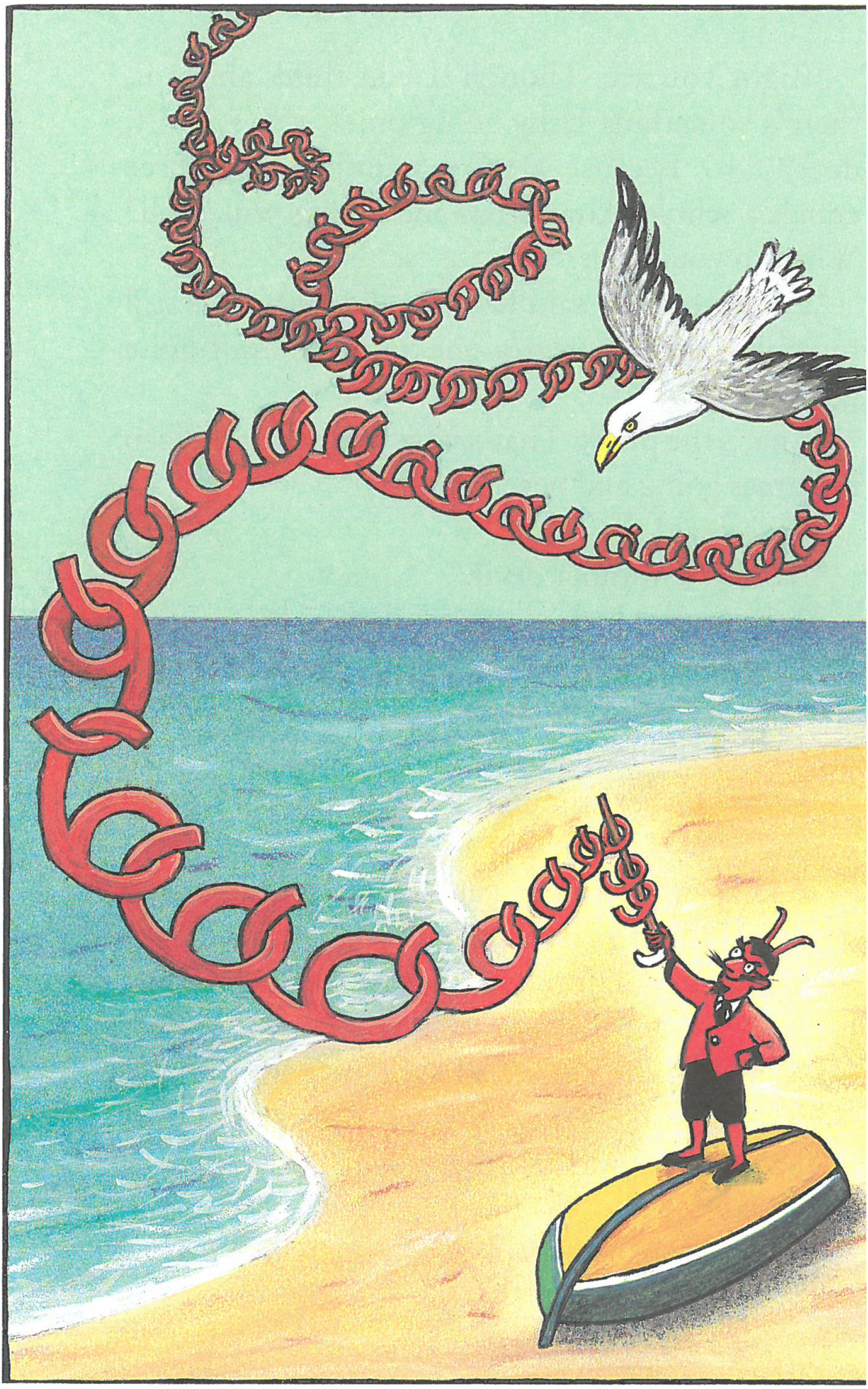
With another little chuckle he waved his walking stick and in the twinkling of an eye the sky was filled with an endless chain of purple nines slithering higher and higher.

"Stop!" Robert shouted. "Please stop! It's making me sick."

"A snap of my fingers and they're gone, but not until you admit that the chain of nines behind the zero, if it goes on forever, will turn out to be equal to one."

Meanwhile the chain had kept growing and the sky slowly darkened with nines. Robert was now as dizzy as he was nauseous, but he refused to give in.

"Not on your life!" Robert shouted. "No matter



The number devil waved his walking stick and in the twinkling of an eye the sky w
filled with an endless chain of purple nines slithering higher and higher.

how many nines you add to your chain, there will always be something missing: the last nine."

"There is no last nine!" the number devil furiously shouted back.

Robert no longer jumped out of his skin each time the number devil lost his temper. By now he knew that whenever it happened there was something interesting coming up, something the number devil couldn't easily explain. But the chain was flapping dangerously close to Robert's head and had wound so tightly round the number devil that much of him had receded from view.

"All right," Robert said. "I give in. But only if you get rid of the nines."

"It's about time," said the number devil, raising his stick, which now had several layers of nines entwined around it. Then he mumbled some gibberish to himself and the chains disappeared in a flash.

"Phew!" said Robert. "Does it only happen with threes and nines, or do other numbers make such awful chains too?"

"There are chains as endless as sand on the beach. How many would you say there are between 0.0 and 1.0?"

Robert thought long and hard.

"An infinite number. As many as between one and till the cows come home."

"Not bad," said the number devil. "Quite good, in fact. But can you prove it?"

"I can."

"Show me."

"All I have to do is write a zero and a dot," said Robert, "and a one after the dot: 0.1. Then a two after that. And so on. If I keep going, I'll have put all the numbers that have ever been written after the dot before I come to 0.2.

"Whole numbers all."

"Of course. All whole numbers. Every number between one and bezillion can have a zero and a dot before it, and every one of them is less than one."

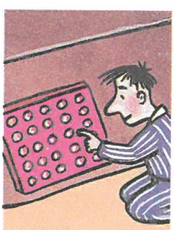
"Splendid, Robert. I'm proud of you."

But proud as he was, he could not leave well enough alone. He had a new idea.

"Many of your numbers after the dot have an interesting life of their own. Would you like to see what I mean?"

"Oh, yes!" said Robert. "Just so long as it won't cram the beach with famous numbers."

"Don't worry. Your big calculator will take care of them. All you need to do is enter seven divided by eleven."



Robert didn't need to be told twice.

7÷11=0,6363636363636…

"Hey, what's up? All those sixty-threes. I bet it goes on forever."

"You bet right. But that's nothing. Try dividing six by seven."

So Robert entered:

6÷7=0,857142857142857…

"The same numbers keep coming back!" he cried. "857142 over and over. It's, like, spinning in a circle!"

"They really are fantastic creatures, numbers! In fact, there's no such thing as an ordinary number. Every one has its own features, its own secrets. You never completely understand what makes them tick. That chain of nines after the zero, for instance: all those nines and still not quite one. And there are many others that behave even worse, that go off the deep end after their zero. They are called the unreasonable numbers, and the reason they're called that is that they refuse to play by the rules. If you have another moment for me, I'll be glad to demonstrate."

Robert knew by now that whenever the number devil made a point of being polite he was in for something. But he was so curious he couldn't resist.

"Fine," he said. "Please do."

"You recall our hopping game, I'm sure. What we did with the two and the five and the ten. Ten times ten times ten equals a thousand, which we write

$$10^3 = 1000$$

because it's faster."

"Right. And when we hop with the two, we get

$$2, 4, 8, 16, 32$$

and so on or—as always with your little tricks—till the cows come home."

"Well then," said the number devil, "how much is two to the fourth?"

"Sixteen! Pretty good, eh?"

"Perfect, my boy! And now let's do the first hop in reverse. Hopping backward, so to speak. Only when you go backward this way, you don't really hop. We call that step 'taking the rutabaga,'

$\sqrt{\ }$

as if we were pulling one of those fine root vegetables out of the ground. So what is the rutabaga of four?"

"Two."

"Right! Taking the rutabaga is the reverse of our first hop. So the rutabaga of a hundred is ten, and the rutabaga of ten thousand is a hundred. What's the rutabaga of twenty-five?"

"Twenty-five," said Robert, "is five times five, which makes its rutabaga five."

"Keep it up, Robert, and you'll be my apprentice someday. Rutabaga of thirty-six?"

"The rutabaga of thirty-six is six."

"Rutabaga of 5,929?"

"Are you crazy or something?" Robert shouted. "How do you expect me to do that one? Mr. Bockel plagues us with enough dumb problems in school. I don't need to dream about them."

"Calm down, calm down," said the number devil. "Little problems like that are what the pocket calculator was made for."

"*Pocket* calculator! The thing's as big as a couch."

"Be that as it may, you'll notice it has a key with this sign on it:

$$\sqrt{}$$

Which means?”

“Rutabaga!”

“Right. Now give it a try.”

$$\sqrt{5929} =$$

Robert did as he was told, and immediately read the following off the backrest of the couch calculator:

77

“Fine. But now hold on to your hat and try the rutabaga of two.”

Again Robert did as he was told, and got the following:

1,4 142 1356237309504880 1688724

“Drat!” he cried. “It’s utter gibberish. Number stew. I can’t make head or tail of it.”

“Nor can anyone else, my dear boy. That’s the point. The rutabaga of two is an unreasonable number.”

irrational

“Is there any way of knowing how it goes on? Because I have a feeling it does.”

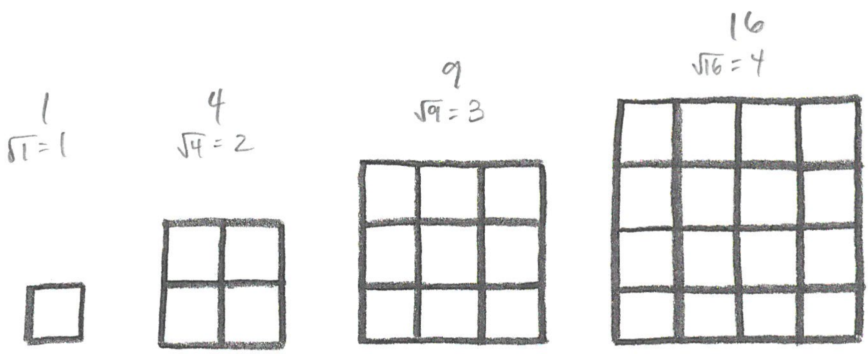
“Right you are, but I’m afraid I can’t help you there. Taking the number any farther would mean running myself, or my calculator, into the ground."

"Wild!" Robert said. "A real monster. But write it like this:

$$\sqrt{2}$$

and butter wouldn't melt in its mouth."

"Well, let's try something a little less daunting." He drew a few figures in the sand and said, "Have a look at these:



Count up the small boxes inside the squares and tell me whether you notice anything special about them."

$$1 \times 1 = 1^2 = 1$$
$$2 \times 2 = 2^2 = 4$$
$$3 \times 3 = 3^2 = 9$$
$$4 \times 4 = 4^2 = 16$$

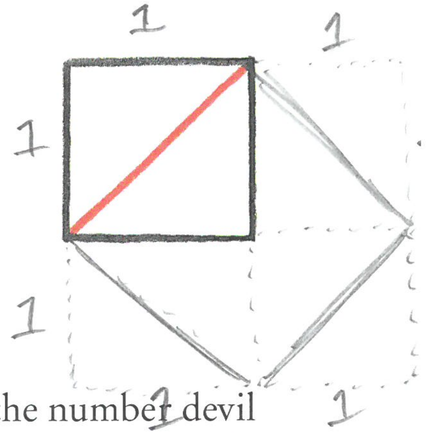
"You bet I do. They're all hopping numbers."

"Right. You see how it works, don't you? Count the number of boxes on the side of each square and you've got the number to hop with. And vice versa. If you know how many boxes the square has—thirty-six, say—and take the number's rutabaga, you get the number of boxes along the side of the square:

$$\sqrt{1} = 1, \sqrt{4} = 2, \sqrt{9} = 3, \sqrt{16} = 4$$

"Great," said Robert, "but what's that got to do with unreasonable numbers?"

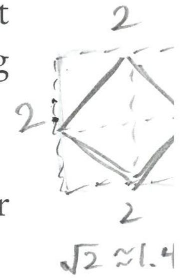
"Well, squares are wily beasties. Never trust a square. They may look innocent, but they can be full of tricks. Take this one, for instance." And he carved a perfectly ordinary empty square into the sand. Then he pulled a red ruler out of his pocket and laid it diagonally across it:



"Now if each side has the length one . . ."

"One what? One inch or one foot?" Robert interrupted.

"It makes no difference," said the number devil impatiently. "One whatever you please. Call it quing or quang for all I care. Now tell me how long the red line is."

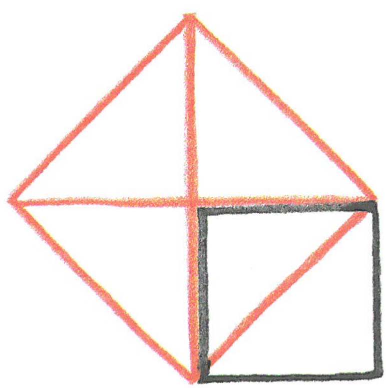


"How should I know?"

"The rutabaga of two!" shouted the number devil triumphantly.

"How did you get that?" asked Robert, who was starting to feel overwhelmed again.

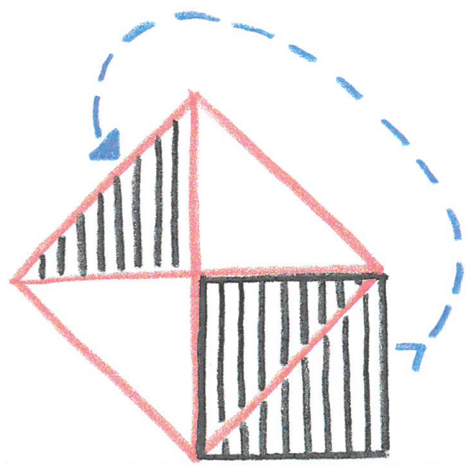
"Don't worry," said the number devil. "We're coming to it. All we have to do is place another square over it at an angle." He pulled five more rulers out of his pocket and laid them in the sand. Which made the figure look like this:



"Now guess how big the red square is, the one at an angle to the black one."

"I have no idea."

"The red square is exactly twice as big as the black one. Shift the lower half of the black one into one of the four corners of the red one and you'll see why."



"It reminds me of a game we used to play when I was little," Robert said. "Heaven and Hell we called it. We'd fold a sheet of paper into sections painted black and red. Open it to black and you went to heaven; open it to red and you went to hell."

"And do you see that in this instance there is twice as much red as there is black?"

"I do."

"Good. Now, since the area of the black square is one times one quang—we agreed to call the length of each side a quang, remember?—we can write it 1^2. And if the red square is twice as large as the black one, what is *its* area?"

"Two times 1^2," said Robert. "In other words, two."

"Correct. Then how long is each side of the red square? I'll give you a hint: all it takes is a backward hop."

"I see!" said Robert, the scales falling from his eyes. "Rutabaga! You need to take the rutabaga of two!"

"Which brings us back to our cockeyed, totally unreasonable number 1.414213 · · ·"

"Stop! Stop!" said Robert quickly. "You'll drive me crazy if you keep on with that number."

"It's not so bad as all that," said the number devil. "But we don't need to work it out. Just don't go thinking that unreasonable numbers are a rarity. Quite the contrary. Take it from me, they're like sand on the beach, more common even than the other kind."

"But there's an infinite quantity of the other kind, the ordinary ones. At least that's what you've been saying. And saying and saying."

"Because it's true. Believe me! It's just that there are many, many more unreasonable ones."

"More than what? More than an infinite quantity?"

"Exactly."

"Now you're going too far," said Robert in no uncertain terms. "I refuse to believe it. More than infinite? Nothing is more than infinite. That's a lot of malarkey, that's what it is."

"Want me to prove it?" asked the number devil. "Want me to conjure up all the unreasonable numbers at once?"

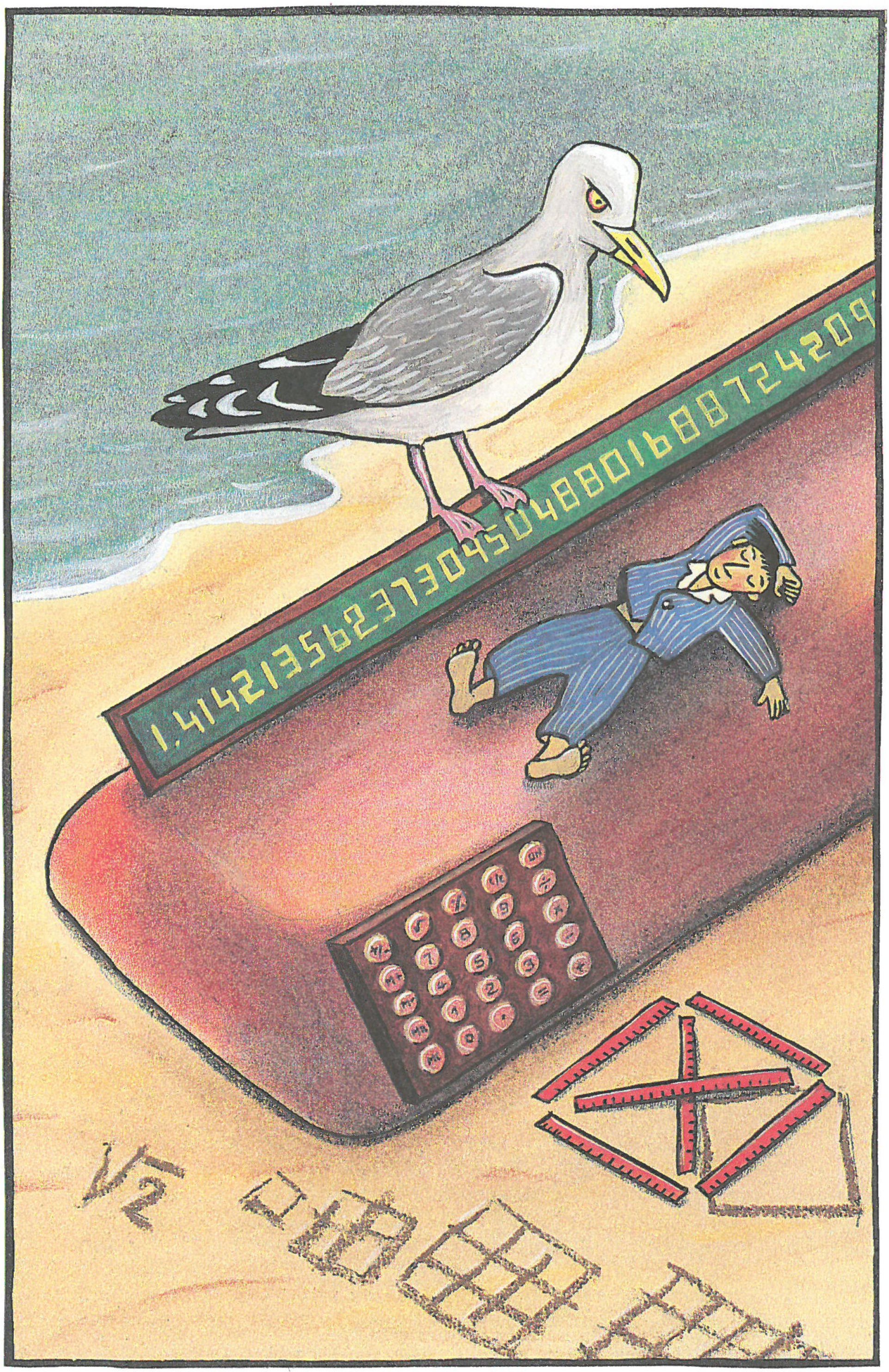
"No, anything but that! The nine chain was bad enough. Besides, what kind of proof is magic?"

"Blast!" the number devil said. "You've got me there!" But he didn't seem terribly annoyed. He merely frowned and started thinking hard.

"I could probably come up with another proof," he said at last, "but only if you insist."

"No, thank you," said Robert. "I've had enough for today. I'm beat. If I don't get a good's night sleep, I'll be in for it tomorrow in school. I think I'll stretch out on your calculator, if you don't mind. It looks awfully inviting."

"Be my guest," said the number devil as Robert lay down on the fleecy, furry, couch-sized calculator. "You're asleep as it is. You learn best when you sleep." And he tiptoed off so as not to awaken him.



"I've had enough for today," said Robert. "I'm beat." And he lay down on the fleecy, furry, couch-sized calculator.

Maybe he's not so bad after all, Robert thought. In fact, he's pretty cool.

Robert slept peacefully, and dreamlessly, late into the morning. He'd completely forgotten the next day was Saturday, and on Saturday, of course, there's no school.