

The Fifth Night





Suddenly it was over. Robert waited and waited for his visitor from the realm of numbers, but the number devil never came. Robert went to bed as usual and most nights he had dreams, but they weren't about couch-sized calculators and hopping numbers. One was about some deep, dark holes he kept stumbling into, another about a junk room full of old suitcases with larger-than-life red ants pouring out of them: the door was locked and he couldn't get out and the ants kept crawling up his legs. In yet another he needed to ford a raging stream, but because there was no bridge he had to jump from rock to rock and just as he was about to reach the other shore all the rocks vanished and he couldn't go forward or back. Nightmares, nothing but nightmares, and no number devil in sight.

At all other times I can decide what I want to think about, Robert said to himself. Only when I

dream do I have to take what comes. Why, I wonder?

“You know what?” he said one evening to his mother. “I’ve come to a decision. From now on—no more dreams for me.”

“Glad to hear it, darling,” she said. “You have trouble concentrating after a bad night, and then you don’t do well in school.”

Of course that wasn’t what bothered Robert in the least, but all he said was “Good night.” He knew you can’t tell mothers everything.

But no sooner did he close his eyes than it all started up again. He was wandering through a desert where there was no shade or water, wearing nothing but a pair of bathing trunks. The heat was terrible. It was just the kind of thing he hated about dreams. On and on he trudged, thirsty, sweaty, with blisters all over his feet, until at last he made out a few trees in the distance.

It must be either a mirage or an oasis, he thought.

So on he hobbled until he came to the first palm. And there he heard a voice calling, a voice he was sure he knew.

“Hello there, Robert!” it called.

He looked up. He was right! It was the number devil, bobbing up and down on the palm leaves. He looked quite at home in the desert.



On Robert hobbled until he came to the first palm. And there he heard a voice calling, a voice he was sure he knew. He looked up. He was right!

“I’m dying of thirst,” Robert called back.

“Come on up,” said the number devil.

Marshaling all the strength he had left, Robert climbed to the top of the tree. His friend greeted him with a coconut and made a hole in the shell with his pocketknife.

The coconut milk tasted delicious.

“Long time no see,” said Robert. “Where have you been?”

“Taking it easy, as you may have noticed.”

“Any plans for today?”

“Don’t you need to rest after your trek across the desert?”

“Oh, don’t worry about me. I’m all right. What’s the matter? No ideas?”

“I’m never at a loss for ideas, my boy.”

“Something to do with numbers, I bet. It’s always numbers.”

“What else? There’s nothing more exciting than numbers. Now what I want you to do is throw your coconut to the ground.”

“Anywhere in particular?”

“No, just down.”

Robert threw the coconut into the sand. From up in the tree it looked like a dot.

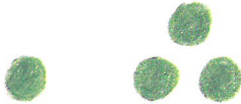
“Now another,” the number devil ordered. “And another. And another.”

“Hey, what’s up?”

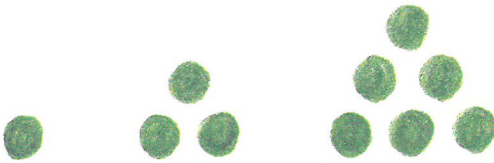


“You’ll see soon enough.”

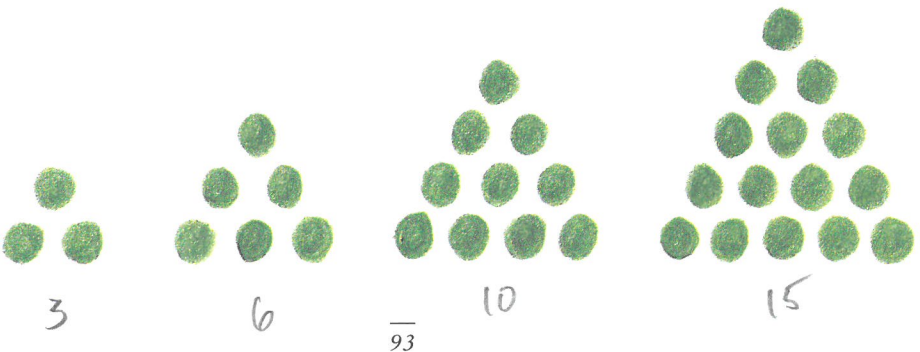
Robert picked three coconuts and threw them all to the ground. This is what he saw in the sand:



“Keep it up,” said the number devil.
So Robert threw and threw and threw.
“What do you see now?”
“Triangles,” said Robert.



“Need any help?” the number devil asked, and the two of them picked and threw, picked and threw until things below looked even more triangular:





“Funny, they fall into such neat patterns,” Robert said. “And I’m not even aiming. I’d never have been able to get them to land like that if I’d tried.”

“Naturally,” said the number devil. “You can only be that accurate in a dream—and in mathematics. Anything in mathematics can make up a nice clean pattern if you put your mind to it. By the way, we didn’t really need coconuts. Tennis balls would have done just fine. Or buttons. Or chocolate-covered cherries. But now, would you count up the number of coconuts there are in each of the triangles?”

“All right. But the first is no triangle at all. It’s just a dot.”

“Or a triangle that has shrunk so small that all you can see of it is a dot.”

“We’re back to our friend one again, I see,” said Robert.

“Next?”

“The second triangle has three coconuts, the third six, the fourth ten, and the fifth . . . I’m not sure. Wait. Let me count.”

“Why? You don’t need to count. You can calculate it.”

“No I can’t.”

“Yes you can. Look, the first triangle, which is so small it doesn’t really count as a triangle,

consists of one coconut. The second has two more—the second row—which comes to:

$$1 + 2 = 3$$

The third has exactly three more—the third row:

$$3 + 3 = 6$$

The fourth has another row with four more:

$$6 + 4 = 10$$

So how many has the fifth one got?"

By this time Robert had no trouble calling it out:

$$10 + 5 = 15$$

“No need to throw down any more coconuts,” he said. “I see how it works. The next triangle would have twenty-one coconuts: the fifteen from triangle number five, plus the six new ones.”

“Good,” said the number devil. “Now we can get down from the tree and make ourselves comfortable.”

The climb down was surprisingly easy, and what did they find waiting for them on the ground but two blue-and-white-striped deck chairs and two glasses of ice-cold orange juice next to a huge swimming pool.

I can see why the number devil picked out this oasis, Robert thought. It's the ideal place for a rest.



“Now we can forget about the coconuts and concentrate on the numbers,” said the number devil when both glasses were empty. “And very special numbers they are too. They’re called triangle numbers, and there are more of them than you think.”

“I had a feeling there’d be a lot of them,” said Robert. “You do like your numbers to go on and on.”

“Well, this time let’s stick to the first ten. Here, let me write them down.”

He got out of the deck chair, picked up his walking stick, leaned over the edge of the swimming pool, and wrote the following in the water:

1 3 6 10 15 21 28 36 45 55 . . .

Sky, sand, water—he doesn't care where he writes, Robert thought, as long as it's numbers he's writing.

"You wouldn't believe the kinds of things these triangle numbers can do," the number devil whispered in his ear. "Think of the differences, for instance."

"The differences between what?"

"Between triangle numbers next to one another."

Robert stared at the numbers floating in the pool and tried to imagine what the number devil had in mind.

1 3 6 10 15 21 28 36 45 55 . . .
2 3 4 5 6 7 8 9 10

* constant second diff.

"Three minus one is two. Six minus three is three. Ten minus six is four. Fifteen minus ten is five. It's just like counting from one to ten! Cool! I bet it goes on like that too."

"Precisely," said the number devil, leaning back contentedly. "But that's not all! Pick a number,

any number, and I can show it to be the sum of either two or three triangle numbers.”

“All right, then,” said Robert. “Fifty-one.”

“Simple! All I need is two—”

$$51 = 15 + 36$$

“Eighty-three.”

“My pleasure—”

$$83 = 10 + 28 + 45$$

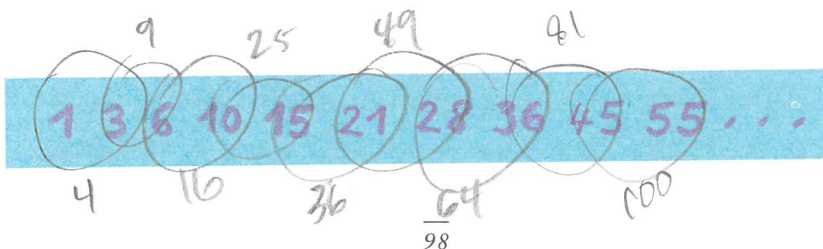
“Twelve.”

“A cinch—”

$$12 = 1 + 1 + 10$$

“See what I mean? It *always* works. And now for something truly sensational. Add each triangle number to the one next to it and you won’t believe what you get.”

Robert stared down at the pool of floating numbers:



And then he started adding:

$$\begin{aligned}1 + 3 &= 4 \\3 + 6 &= 9 \\6 + 10 &= 16 \\10 + 15 &= 25\end{aligned}$$

“Why, they’re all hopping numbers: 2^2 , 3^2 , 4^2 , 5^2 .”

“Pretty good, eh? And you can go on and on.”

“I believe you,” said Robert. “I’d rather have a swim.”

“Fine, only first let me show another one of my tricks.”

“But I’m hot,” Robert grumbled.

“All right then. I’ll be on my way.”

Now he’s hurt again, thought Robert, and if I let him go, I’ll probably end up dreaming about those red ants again. So he said, “No, no! Do show it to me!”

“Aha! You’re curious.”

“Yes, yes.”

“Then pay close attention. What do you get if you add up all the ordinary numbers from one to twelve?”

“Hey, that’s not like you at all! That’s the kind of thing Mr. Bockel would ask.” Robert glanced over at the number devil with alarm.

“Have no fear. It’s simple as pie with triangle numbers. Just go to the twelfth number and you’ve got the sum of the numbers from one to twelve.”

Robert looked down at the water again and counted:

sum of 1 to 9? ↓

1 3 6 10 15 21 28 36 45 55 66 78 ...

“Seventy-eight,” he said.

“Right.”

“How come?”

The number devil picked up his stick and wrote:

1 2 3 4 5 6
12 11 10 9 8 7

“All I’ve done is write the numbers from one to twelve in two rows, the first six from left to right and the second six from right to left. Draw a line under them and add them up.

$$\begin{array}{cccccc}
 1 & 2 & 3 & 4 & 5 & 6 \\
 12 & 11 & 10 & 9 & 8 & 7 \\
 \hline
 13 & 13 & 13 & 13 & 13 & 13 = 78
 \end{array}$$

What do you get?"

"Six times thirteen," said Robert.

"I hope you don't need a calculator for that."

"Six times thirteen," said Robert, "is seventy-eight. The twelfth triangle number!"

"See what triangle numbers are good for? And by the way, quadrangle numbers aren't bad either."

"I thought we were going for a swim."

"We can swim later. First the quadrangle numbers."

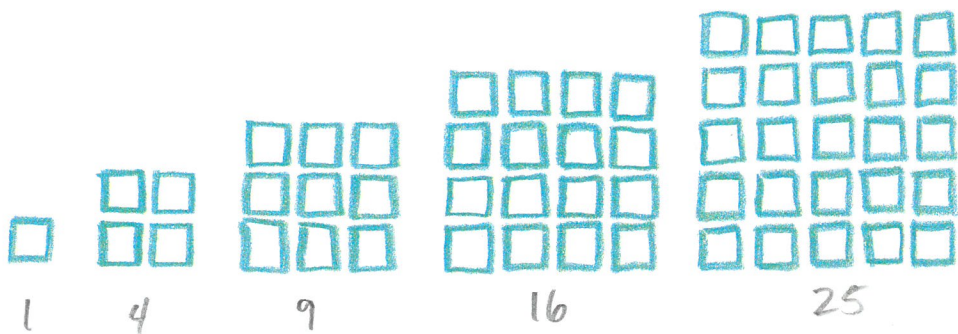
Robert glanced longingly at the swimming pool, where the triangle numbers were bobbing up and down like ducklings.

"If you keep on like this, I'll wake up and all the numbers will disappear."

"The numbers *and* the pool," said the number devil. "But you can't just stop dreaming whenever you please. And who's boss here, anyway? You or me?"

There he goes again, thought Robert. Now he's losing his temper. Soon he may even start yelling at me. Sure it's all a dream, but I never like being yelled at, not even when I'm asleep. Heaven knows what he's got up his sleeve this time.

The number devil took some ice cubes from a cooler and set them out on a table in five perfect squares, each one larger than the other. "It won't take long," he said to make Robert feel better. "It's the same as with the coconuts, only this time we're using squares instead of triangles."



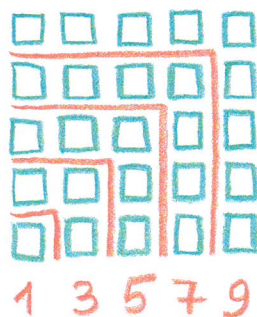
"You don't have to say another word. It's perfectly clear. They're hopped, every one of them. All I have to do is count the number of cubes on one side and hop the result:

$$\begin{aligned}
 1 \times 1 &= 1^2 = 1 \\
 2 \times 2 &= 2^2 = 4 \\
 3 \times 3 &= 3^2 = 9 \\
 4 \times 4 &= 4^2 = 16 \\
 5 \times 5 &= 5^2 = 25
 \end{aligned}$$

And so on, as usual.”

“Very good,” said the number devil. “Devilishly good. No, I’ve got to hand it to you. You’re a top-notch apprentice.”

If you’re not as hot as Robert, you might want to play with the cubes a bit before they melt. Just divide up the square like this:



The numbers indicate the number of cubes within the lines you have drawn. What do you get if you add them together? The answer will look quite familiar.

“Now can I have my swim?” Robert moaned.

“But surely you want to learn how the pentagonal numbers work. And the hexagonals.”

“Oh, no, thank you,” said Robert. “Really!”
And he stood up and dived into the water.

“Wait, wait!” the number devil called out. “The pool is full of numbers. Wait until I fish them out.”

But by then Robert was swimming his way through them and they were bobbing all around him, and he swam and he swam until he couldn't hear what the number devil was saying. It was a pool that went on and on just like the numbers, and it was just as exciting.

