

The Sixth Night





“You probably think I’m the only one,” said the number devil the next time he turned up, perched on a folding chair in the middle of a vast potato field.

“The only what?” asked Robert.

“The only number devil. But I’m not. I’m one of many. Number Heaven, where I come from, is teeming with us. I’m not even one of the bosses. The bosses do nothing but sit and think. Now and then one of them will laugh and say something like ‘ R_n equals h_n factorial times f of n open bracket a plus θ close bracket,’ and the others nod and laugh along. There are times when I don’t understand a thing.”

“You poor devil,” Robert said. “Here I thought you were so sure of yourself.”

“Why do you think they send *me* out at night? Because the bigwigs have things to do other than visiting apprentices like you.”

“So I’m lucky to have even you. Is that what you’re saying?”

“Don’t get me wrong,” said Robert’s friend (because they were pretty much old friends by now). “I have nothing against what they cook up, the bosses up there in Number Heaven. One I particularly like is a fellow named Bonacci, an Italian, who sometimes lets me in on what he’s doing. He’s been dead for years now, poor Bonacci, but that doesn’t matter when you’re a number devil. Besides, he’s a fine chap, and was one of the first to understand what zero means. He didn’t discover it, mind you, but he did come up with what we call Bonacci numbers. A capital idea! And like most good ideas, it begins with—what do you think?—a one. Or, rather, two ones: $1 + 1 = 2$.

You take the last two numbers and add them together,

and ...
keep ...
going ...
down ...
the line ...”

1 = 1
1 + 1 = 2
1 + 2 = 3
2 + 3 = 5
3 + 5 = 8
5 + 8 = 13
8 + 13 = 21

“Till the cows come home?”

“You guessed it.”

Next the number devil started running through the Bonacci numbers in a kind of singsong. The aria from a Bonacci opera, you might say.

“Oneonetwothreefiveeightthirteentwentyone thirtyfourfiftyfiveeightynineonehundredandforty fourtwohundredandthirtythreehundredand seventyseven . . .”

Robert clapped his hands over his ears.

“All right, I’ll stop,” said the number devil, “though I’d better write them out so you can see what they look like.”

“What have you got to write on?”

“What would you like? How about a scroll?”

Unscrewing the tip of his walking stick, he pulled out a thin roll of paper, tossed it on the ground, and gave it a poke. An endless stream of paper rolled out along a furrow.

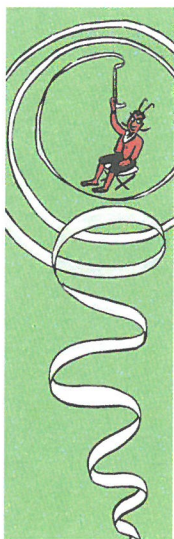
How could all that paper fit into the stick? Robert wondered.

Meanwhile on and on it rolled until it rolled out of sight with all its Bonacci numbers:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13
1	1	2	3	5	8	13	21	34	55	89	144	233
+0	+1	+1	+2	+3	+5	+8						

$$\begin{array}{r} 2^2 = 4 \\ 2^2 = 9 \\ \hline 2^2 = 1.2 \end{array}$$

$$\begin{array}{r} 109 \\ 2 \\ \hline 8 \\ 17^2 = 169 \end{array}$$



After that the numbers were so far off and tiny that Robert couldn't read them.

"Now what?"

"If you take the sum of the first five and add one, you get the seventh. If you take the sum of the first six and add one, you get the eighth. And so on and so forth."

"I see," said Robert, but he didn't sound particularly excited.

"It also works if you jump over numbers. Keep in mind though: the first one must always be present.

You start like this:

$$1 + 1 = 2$$

(and now you jump one)

$$+ 3$$

(and now you jump another)

$$+ 8$$

(and now you jump yet another)

$$+ 21$$

And what do you get when you add them up?"

"Thirty-four," said Robert.

"In other words, the Bonacci number after twenty-one. And if that's too hard for you, you can get there by hopping. For example, you take Bonacci number four, which is three, and you make it hop: 3^2 . Which is . . ."

“Nine,” said Robert.

“Then you take the next Bonacci number, number five, which is five, and make *it* hop.”

“ $5^2 = 25$,” said Robert without missing a beat.

“Good, and now add the two together.”

$$9 + 25 = 34$$

“Another Bonacci number!” he cried.

“And not only that,” the number devil said, rubbing his hands. “The ninth. Because four and five make nine.”

“Fine, fine. Fine and dandy. But tell me, what are they good for, your Bonacci numbers?”

“You don’t think mathematics is for mathematicians only? Nature needs numbers too. Trees add. Fish subtract.”

“Come on,” said Robert. “You don’t expect me to believe that.”

“I expect you to believe that every living thing uses numbers. Or at least behaves as if it did. And some may well have an understanding of how they work.”

“Well, I don’t believe it.”

“Take rabbits, for instance. They’re more lively than fish. I bet there are rabbits all over this potato field.”

“I don’t see any,” said Robert.

“Look, there are two now!”

Sure enough, two teensy white rabbits hopped up to Robert and plonked themselves at his feet.

“A male and a female, I think,” said the number devil. “And a male and a female makes *one* couple. As you know, *one* is all we need to start things rolling.”

“He wants me to believe you can do arithmetic,” Robert said to the rabbits. “Well, I’m too smart for that.”

“What do you know about rabbits, Robert?” said the two rabbits with one voice. “I bet you think we’re snow rabbits.”

“Snow rabbits?” said Robert, who wanted to show them that he did know something about rabbits. “Snow rabbits are winter animals, aren’t they?”

“Correct,” they replied, “and they’re always white, while we’re white only when we’re young. It takes us a month to grow up, and then our fur turns brown. And then we want to have babies. It takes another month for them to be born. One boy bunny and one girl bunny.”

“Just two?” asked Robert. “I always thought rabbits had oodles of bunnies.”

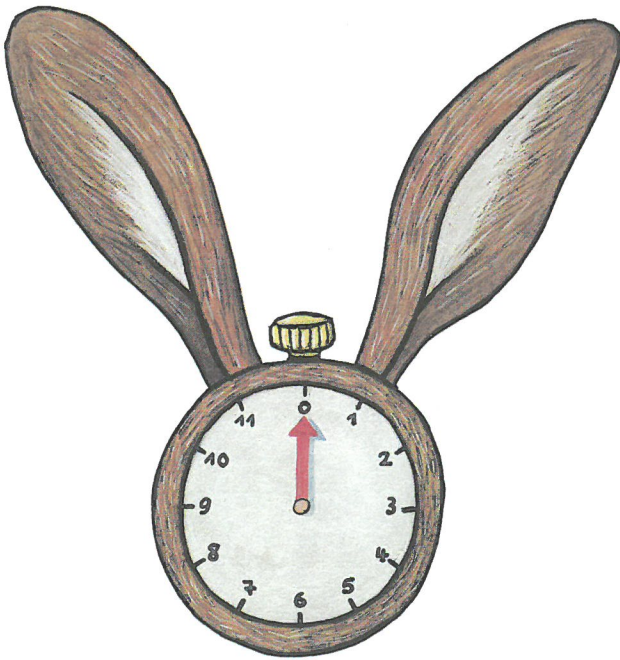
“We have, we have,” said the rabbits, “but not all at once. Two a month is enough. And they grow up and do the same. You’ll see.”

“But I’ll have long since woken up before then. I have to go to school tomorrow morning . . . ”

“No problem,” the number devil interrupted. “Time runs faster here in the potato field. A month lasts only five minutes. At least when you use the special rabbit clock I just happen to have with me.”

With these words, he pulled a large pocket watch out of his trouser pocket. It had two large rabbit ears, but only one hand.

“The hand shows months, not hours,” he said, “and a bell rings every time a month goes past. All



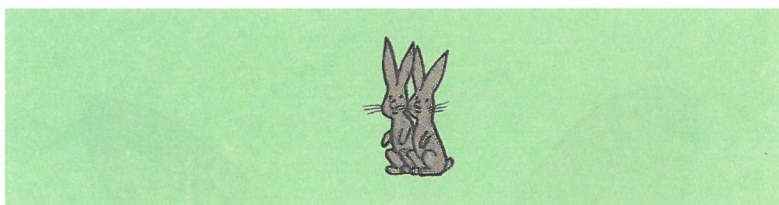
I have to do to set it in motion is to press the button on top. Shall I?”

“Oh, do!” the rabbits cried.

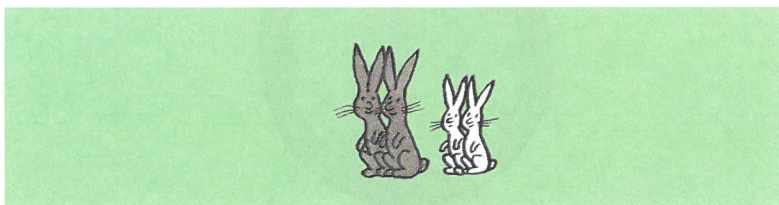
“Good.”

The number devil pressed the button. The clock started ticking; the hand started moving. When it reached one, the bell rang. A month had passed, the rabbits were much bigger and their fur had changed color—from white to brown.

When the hand reached two, two months had passed, and the mother rabbit had brought two teeny-weensy white rabbits into the world.



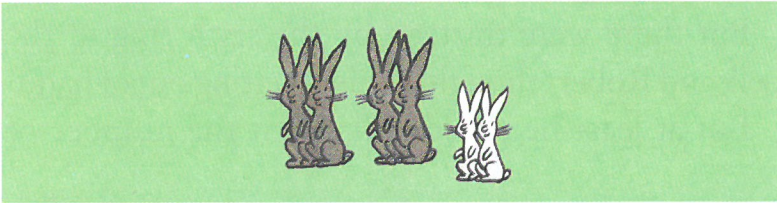
Now there were two couples, one younger, one older. But they did not remain satisfied for long. They wanted more babies, and by the time the hand had reached three and the bell rang again,



mother rabbit had given birth to the next two rabbits.

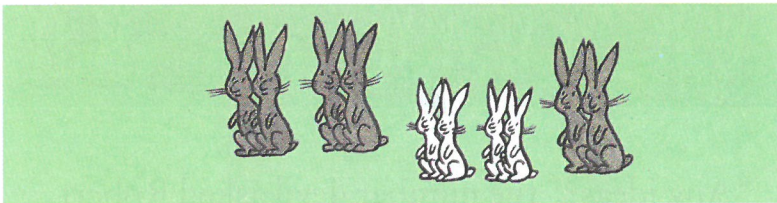
Robert counted the couples. Now there were three: the original one (brown), the children from their first litter, who had meanwhile grown up (and turned brown), and the furry white babies.

When the hand reached four, the old mother



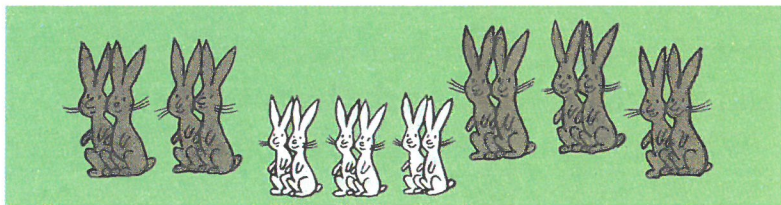
rabbit gave birth to two more rabbits, and her first daughter, not to be outdone, gave birth to two. That meant that there were now five couples hopping around the potato field, three of which were brown and two of which were white.

“I wouldn’t try to keep them straight if I were



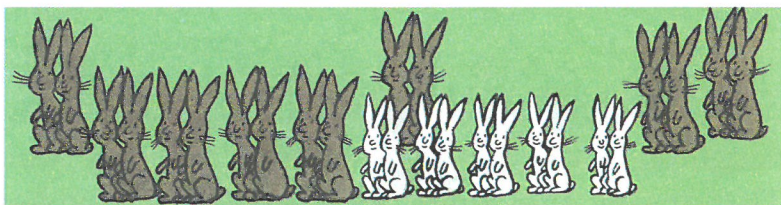
you,” said the number devil. “You’re going to have a hard enough time just counting them.”

Robert had no trouble when the clock reached five. There were only eight couples.

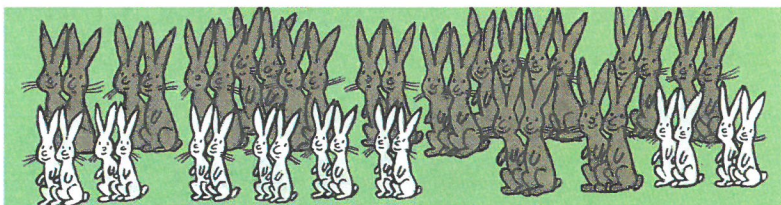


But there were thirteen by the sixth ring of the bell, and Robert thought, This is getting out of hand!

But at least he could still count the number of



couples when the clock reached seven. There were exactly twenty-one.



“Any ideas?” the number devil asked Robert.

“Of course. It’s obvious,” Robert answered.

“Bonacci numbers all the way:



The rabbit clock ran on and on. "Help!" Robert shouted. "Thousands of rabbits and no end in sight! This is no joke, it's a nightmare!"

1, 1, 2, 3, 5, 8, 13, 21 ...

But even as he spoke, new hordes of rabbits were being born and joining their brown and white kin romping over the potato field. Before long Robert couldn't keep up with them anymore. And the rabbit clock ran on and on.

"Help!" Robert shouted when the hand started in on its second round. "Thousands of rabbits and no end in sight! It's awful!"

"Let me show you the rabbit list I've put together. Then you can see the whole picture. It shows everything that goes on between zero and seven."

"But it's long past seven," Robert said. "There must be thousands of them."

"4,181 couples. Which means that in five minutes there will be 6,765."

"Do you plan to let them go on? Because if you do, the whole earth will soon be covered with rabbits."

"Sooner than you think," said the number devil without batting an eyelid. "All it will take is a few rounds of the clock."

"Well, stop them! Please!" Robert begged. "This is no joke, it's a nightmare! Look, I've got nothing against rabbits. I *like* rabbits. But enough is enough. Stop them! Please!"

Rabbit
Clock






























Parents

Children

Grandchildren

Great-
grand-
children

Bonacci
Numbers

					1
					1
					2
					3
					5
					8
					13
					21

“All right, Robert. But only if you admit that the rabbits are behaving as if they had learned their Bonacci numbers by heart.”

“Fine. Great. Anything you say. I admit it. Just stop them or they’ll be crawling all over us in no time!”

The number devil pressed the button on the top of the rabbit clock two times, and the clock started running backward. Each time the bell rang a large number of rabbits vanished and after a few turns of the hand there were only two rabbits left in the potato field.

“What about these?” asked the number devil. “Do you want to keep them?”

“I don’t think so. They’d just start over again.”

“Yes, that’s nature!” said the number devil, rocking gleefully on his folding chair.

“And that’s how Bonacci numbers are,” Robert responded. “I don’t know if I like the way they take off for infinity.”

“Though, as you’ve seen, they can just as easily go the other way. We’re back where we started from. With our one.”

And so again they parted in peace, leaving the rabbit couple to their own devices. The number devil went back his old friend Bonacci and their number-crunching cronies in Number Heaven.

Robert slept dreamless through the night. When the alarm rang the next morning, he was relieved to see it came from a perfectly ordinary alarm clock and not a rabbit clock.



If you still don't believe that nature acts as if it knew how numbers work, turn to the tree on the next page. Maybe you found all those rabbits a bit confusing. Well, a tree can't go romping through a field, so you won't have any trouble counting its branches. Start from below, at the red line numbered one. It runs only through the trunk, as does line two. One line higher, at line three, the trunk has been joined by a branch. Keep going. How many branches are there by the time you reach the top, line nine?

9 34
8 21
7 13
6 8
5 5
4 3
3 2
2 1
1 1

