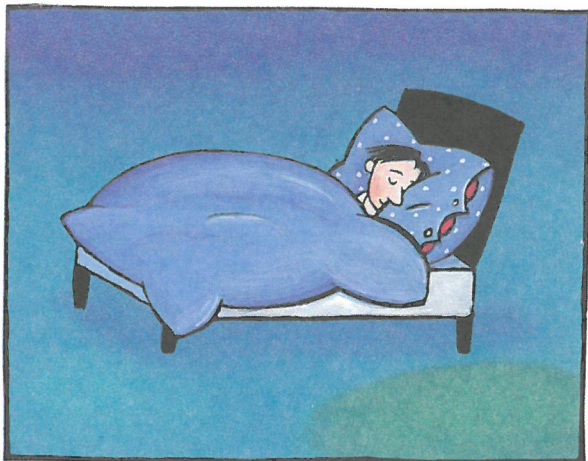


The Seventh Night





“I’m terribly worried,” said Robert’s mother. “I don’t know what’s wrong with the boy. He used to spend all his time in the park playing ball with Al and Charlie and Enrique. Now he shuts himself up in his room and spends all his time painting rabbits, rabbits, and more rabbits.”

“Quiet, Mother. Please!” Robert said. “I can’t concentrate.”

“And the numbers he keeps muttering to himself. Numbers, numbers, and more numbers. It’s not normal.”

She was talking out loud to herself, as if Robert weren’t in the room.

“He didn’t used to be interested in numbers. You should have heard him go on about his teacher and the problems he gave him to do.” Finally she turned to Robert and said, “Isn’t it time you got some fresh air?”

Robert looked up from his painting and said,

“You’re right. If I keep counting rabbits I’m going to get a headache.”

So off he went to the park, a large grassy place with not a rabbit in sight.

“Hi!” Al called out when he caught sight of Robert. “Want to play?”

Enrique, Gary, Hugh, and Jamil were there too. They were playing football, but Robert didn’t feel like joining in. They have no idea how trees grow, he thought.

It was time to eat when he got home, and he went straight to bed after supper, slipping a thick-tipped felt pen into his pajama pocket, just in case.

“Since when do you go to bed so early?” his mother asked. “You always used stay up till all hours of the night.”

But Robert knew exactly what he had in mind and had no intention of telling his mother about it. She’d never believe him if he told her that rabbits, trees, and even fish understand how numbers work and that he had made friends with a number devil.

And no sooner did his head hit the pillow than the number devil was on the scene.

“Today I have something extraordinary to show you,” he said.

“Anything you like, just no more rabbits. They

tortured me to death all day. I couldn't keep track of the whites and the browns."

"Forget it and come with me."

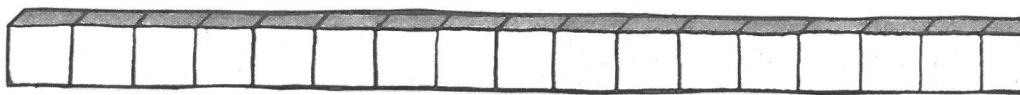
He took Robert to a white house in the form of a cube. The inside was white too, even the staircase and the doors.

"There's no place to sit," Robert complained when they went into a large, bare, completely white room. "And what are those stones doing over there?"

But when he went up to the tall pile of objects in one corner and looked at them more carefully, he realized they weren't stones at all. "They seem to be large cubes of glass or plastic," he said, "with something glittering inside. Something electric."

"Electronic," said the number devil. "What do you say we build a pyramid?" He took a few cubes and laid them out in a row along the white floor. "Well, what are you waiting for?"

Working together, they laid the following row:



"Stop!" the number devil called out suddenly. "How many cubes have we got now?"

Robert counted them up.



“Seventeen,” he said, “an unexciting number.”

“More exciting than you imagine. Subtract one, for instance.”

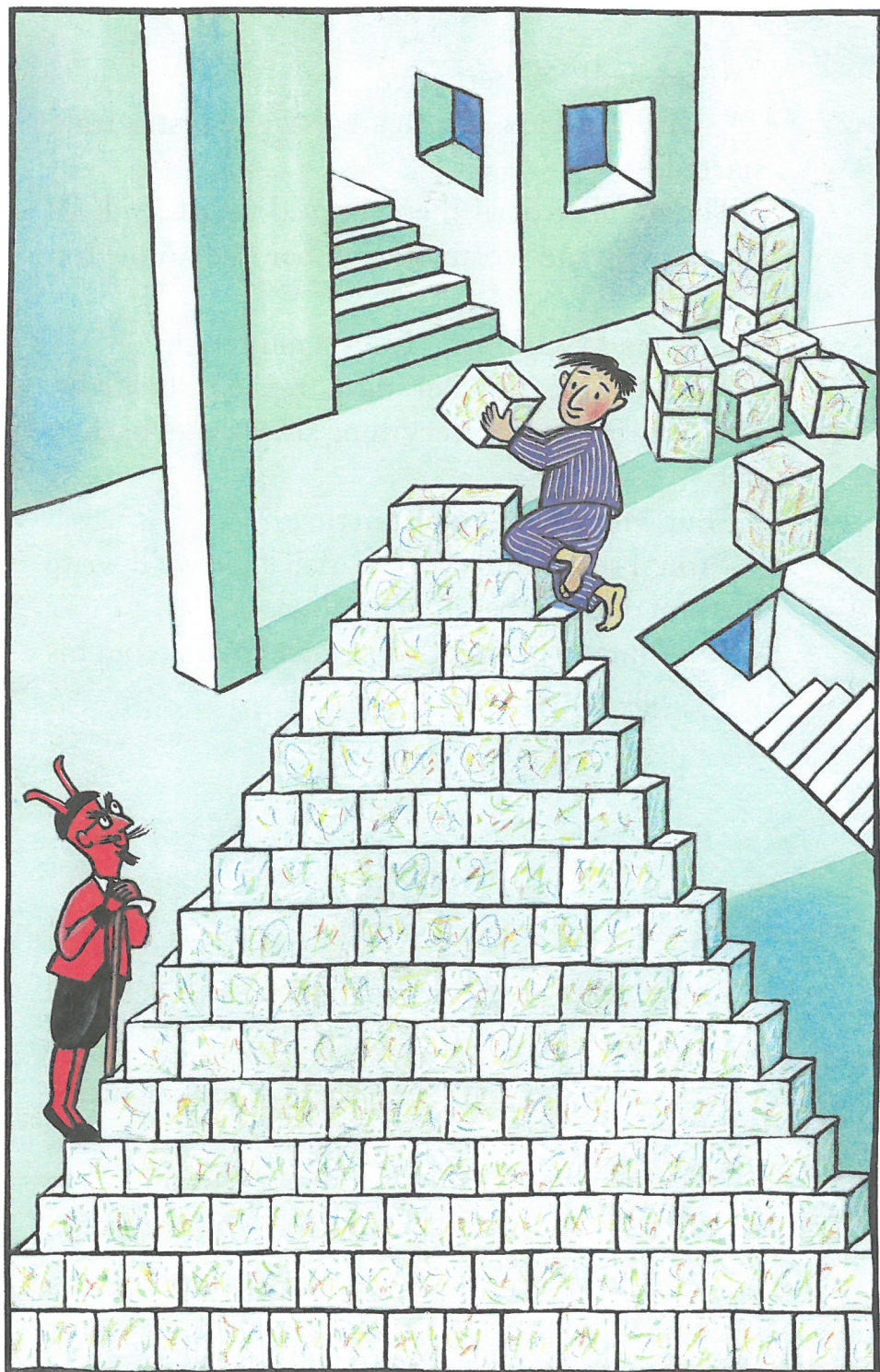
“And you get sixteen. A hopping number. A two that’s been made to hop four times: 2^4 .”

“Good for you,” said the number devil. “You’re getting very observant. But let’s get back to work. Each cube in the next row goes on a crack between two cubes in the first row, the way bricklayers build a wall.”

“Okay,” said Robert, “but it’ll never be a pyramid. Pyramids are triangular or rectangular at the base, and this thing is flat. It won’t be a pyramid, but it can be a triangle.”

“Fine,” said the number devil. “Then we’ll build a triangle.” Which is what they did.





“They seem to be large cubes of glass or plastic,” Robert said, “with something glittering inside. Something electric.”

“Finished!” Robert cried.

“Finished? How can that be? We’re just getting started.”

The number devil then climbed up one side of the triangle and wrote the number one on the top cube.

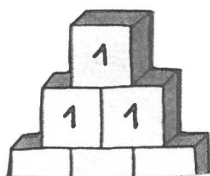
“You and your ones,” Robert muttered.

“Right,” the number devil replied spiritedly. “Because in the end everything always goes back to one.”

“But where do we go from there?”

“You’ll see, you’ll see. On each cube we’ll write the sum of the cubes directly above it.”

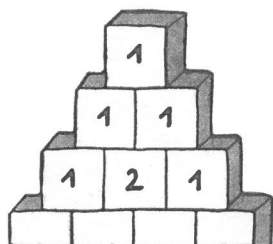
“Nothing to it,” said Robert, and pulling out his trusty felt pen he wrote:



“All ones,” he said. “No need for the calculator yet.”

“Not *just* yet,” said the number devil. “Proceed.”

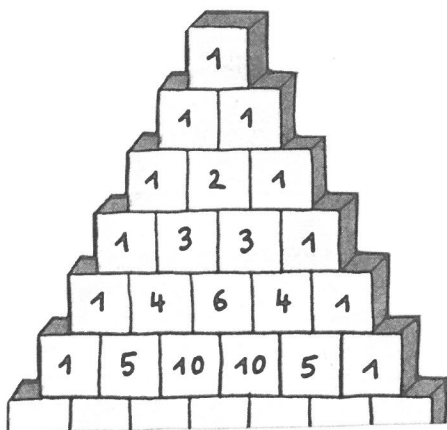
And Robert wrote:



“Child’s play,” he said.

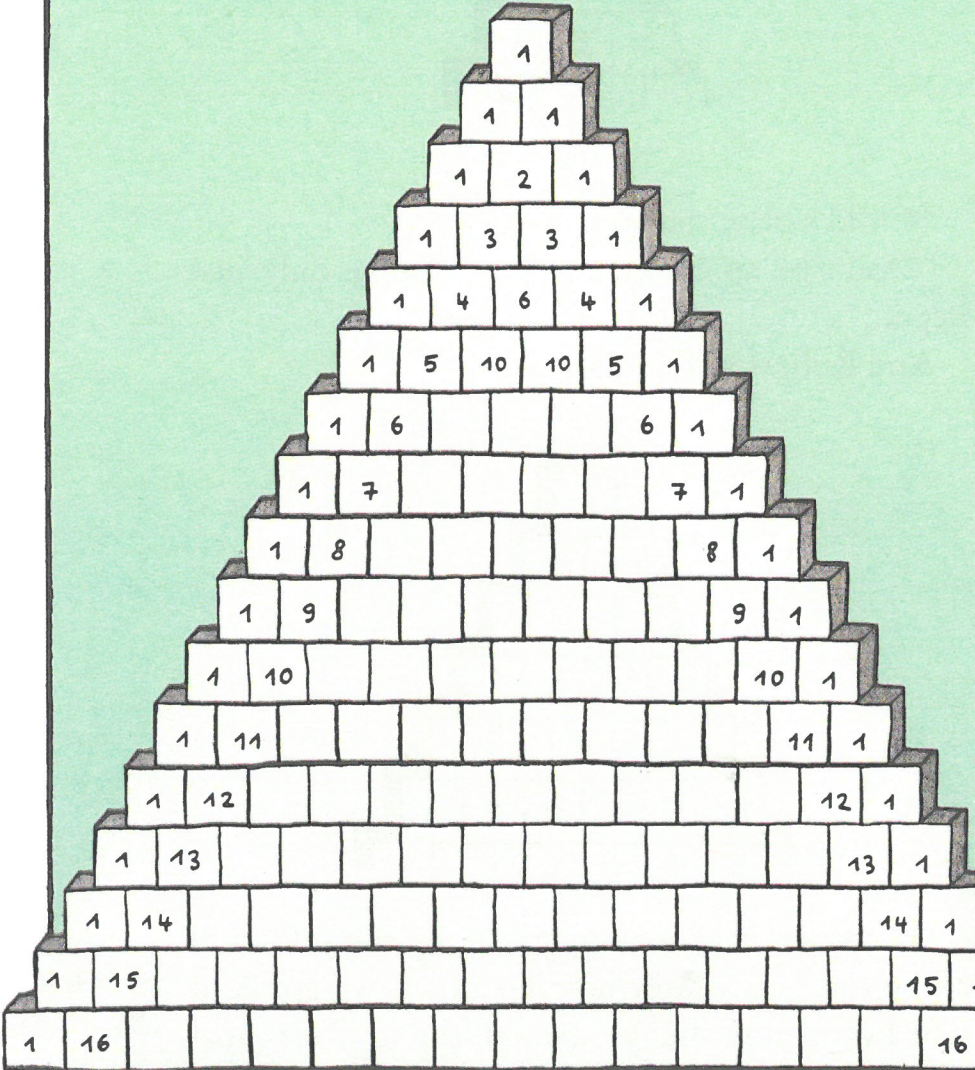
“Don’t be so cocky, my boy. You’ve only just begun.”

And Robert wrote:



“I can see that the numbers along the sides will all be ones no matter how far down we go. And that I can fill in the numbers in the next diagonal rows on either side without doing the arithmetic because they’ll just be the perfectly normal numbers: 1, 2, 3, 4, 5, 6, 7 . . .”

He climbed up and down the triangle writing:



“What about the next diagonal row, the one right next to the 1, 2, 3, 4, 5, 6, 7 . . . ? Read out the first four numbers.”

A knowing smile came over the number devil’s face as Robert read down the row from right to left.

“1, 3, 6, 10 . . . Hey, they look familiar.”

“Coconuts!” cried the number devil.

“Right, right! Now I remember. One, three, six, ten—the triangle numbers!”

“And how do you make them?”

“Sorry. That I don’t remember.”

“Simple.”

$$1 + 2 = 3$$

$$3 + 3 = 6$$

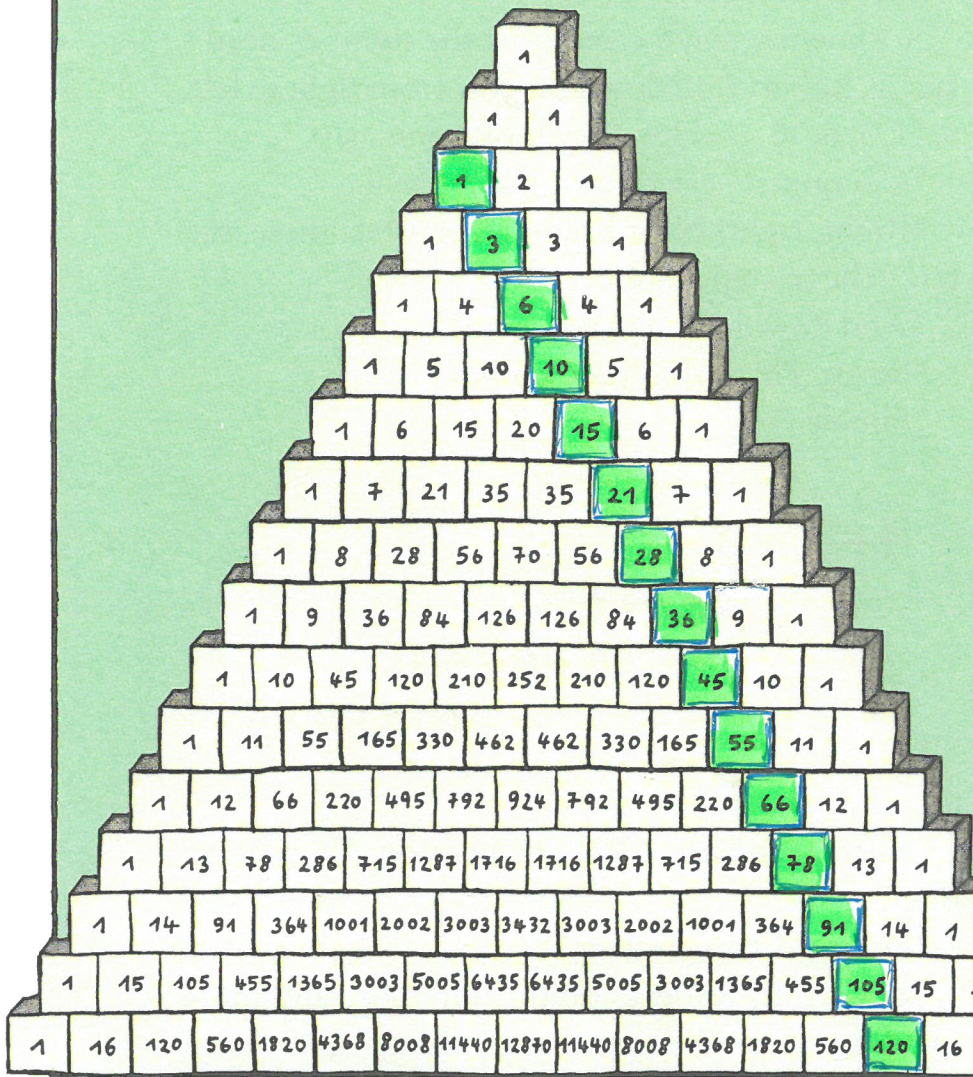
$$6 + 4 = 10$$

$$10 + 5 = 15$$

“Fifteen and six,” Robert went on, “is twenty-one.”

“So you do remember!”

As a result, Robert could fill in more and more numbers. On the one hand, things got easier, because he lowered himself closer and closer to the ground; on the other hand, the numbers got awfully long awfully fast.



“Hey, you can’t expect me to do that kind of addition in my head.”

“Don’t get all worked up now,” said the number devil. “I wouldn’t be much of a number devil if I couldn’t take care of this in a flash.”

And in a flash he had filled in the entire triangle.

“You really had to squeeze in that 12,870!” said Robert.

You can say that again! You may think this is only good for giving you a headache. Wrong! Quite the contrary, in fact. It’s good for lazy-bones who don’t want to bother with long sums. Let’s say you need to find the sum of the first twelve triangle numbers. All you have to do is run your finger down the third diagonal row—the one that goes 1, 3, 6, 10, and so on—until you come to the twelfth cube. Then find the number just below it and toward the center. What is it?

By so doing, you have saved yourself the effort of working out what $1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 45 + 55 + 66 + 78$ comes to.

“Oh, that’s nothing. There’s lots more to the triangle than that! Have you any idea what we’ve built?” the number devil then asked. “It’s more than a triangle. It’s a monitor, a screen. Why do you think all the cubes have electronic insides? All I have to do is turn it on and it will light up.”

With one clap of the hands he turned out the lights and with another he lit the cube on top, lit it bright red like a traffic light.

“There’s that one again,” said Robert.

At the next clap of his hands the first line went out and the second line glowed red.

“Would you mind adding them up for me?” the number devil asked.

“ $1 + 1 = 2$,” Robert mumbled. “Big deal!”

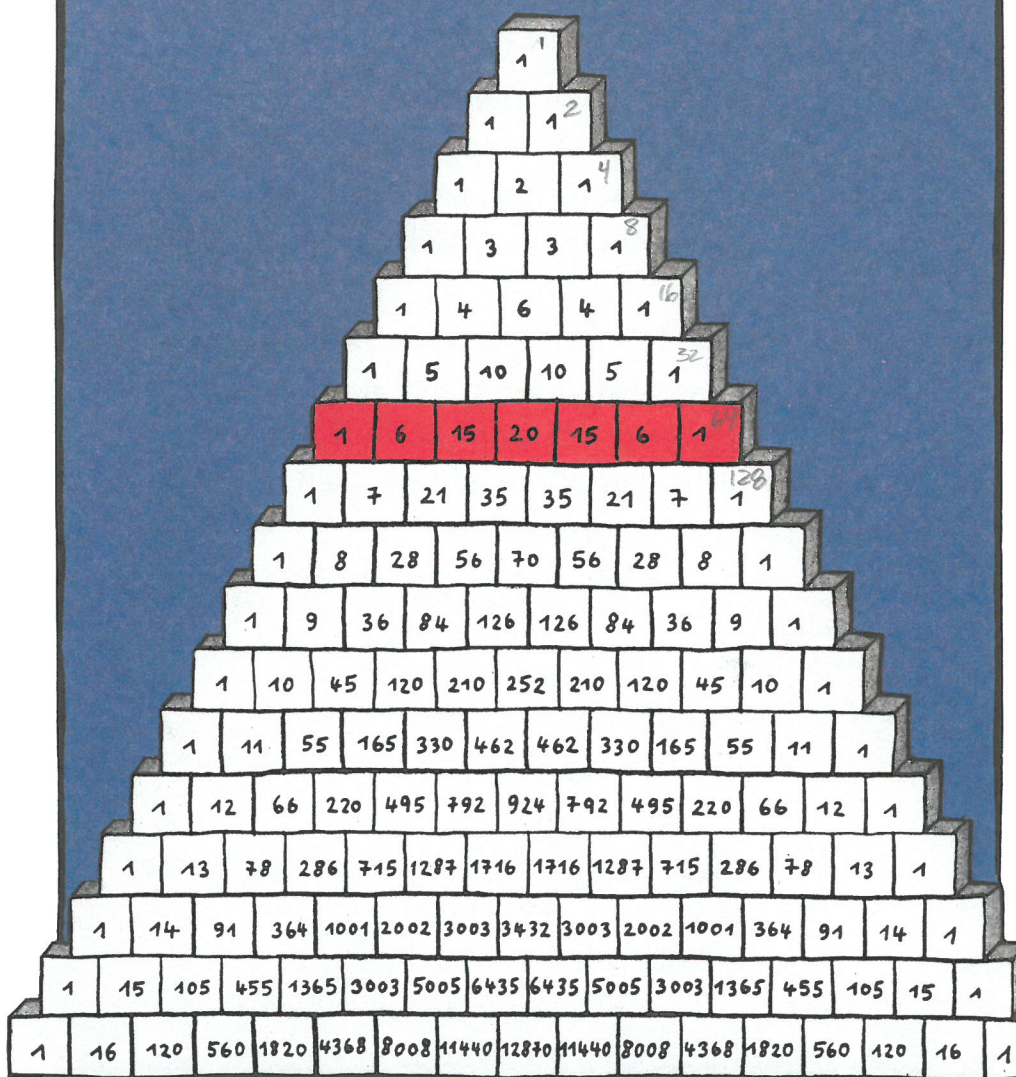
The number devil clapped his hands again, and now the third line shone red.

“ $1 + 2 + 1 = 4$,” said Robert. “I get it, I get it. You can stop clapping. It’s our old friends, the hopping twos. The next line will be $2 \times 2 \times 2$, or 2^3 , in other words, eight. And so on down the line: sixteen, thirty-two, sixty-four. Until we come to the bottom of the triangle.”

“The last line is 2^{16} ,” said the number devil, “and that’s quite a hefty number: 65,536 in case you’re interested.”

“I could do without it.”





“Fine, fine,” the number devil said, clapping his hands, and all of a sudden it was dark again. “Are you up to a visit from some more old friends?” he asked.

“It depends on who they are.”

The number devil clapped three times, and the cubes instantly lit up again, though this time some were orange, others blue, and yet others green or red.

“Looks like a Christmas tree,” said Robert.

“Do you see the color-coded stairs leading from the top right to the bottom left? What do you think will happen if we add up each color? Start with the red one on top.”

“Which is all by itself,” said Robert. “One, as always.”

“Now the yellow one just below it.”

“All by itself. One.”

“Now we come to the blue. Two cubes.”

“ $1 + 1 = 2$.”

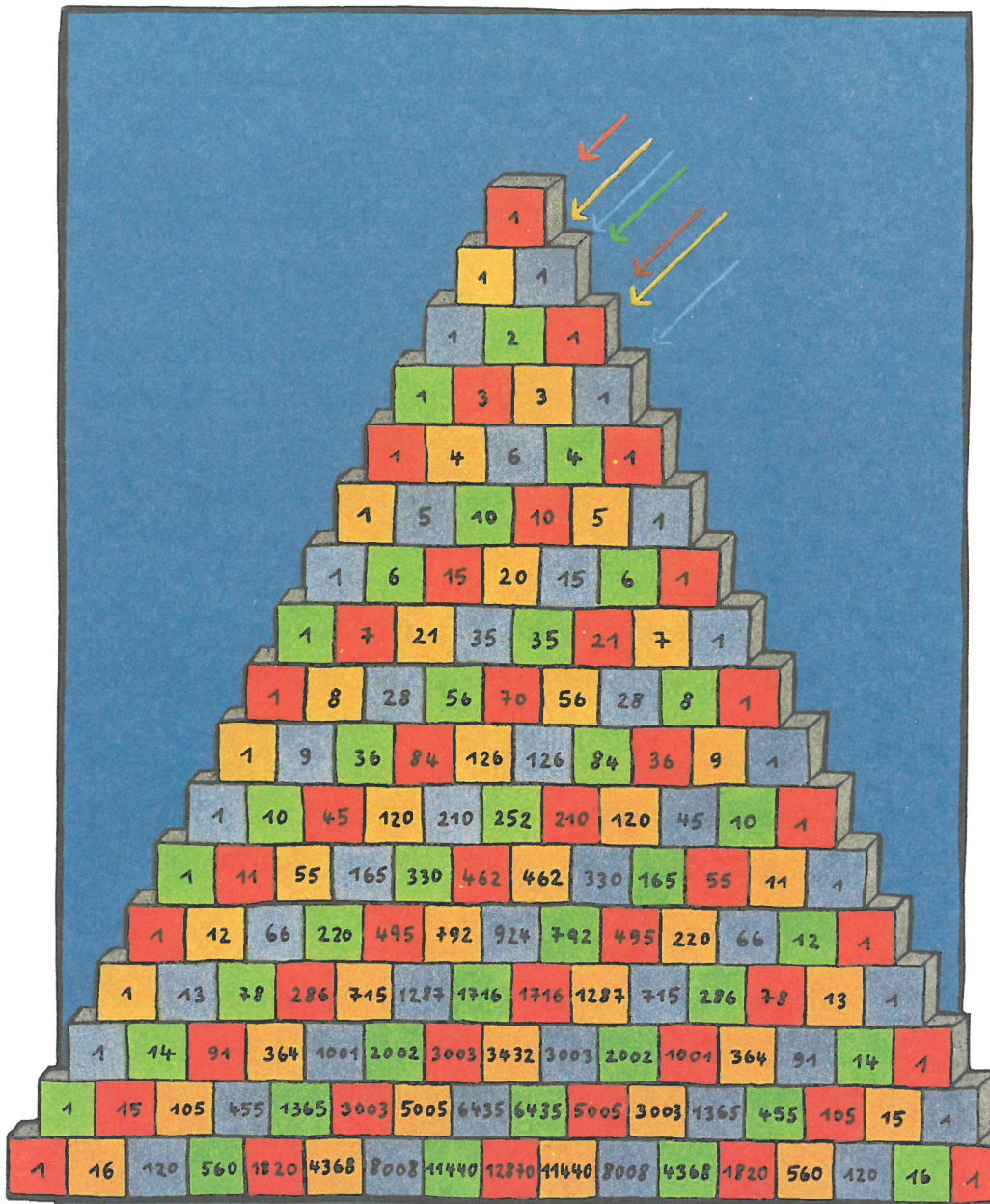
“Then the green just below it. Two green cubes.”

“ $2 + 1 = 3$.”

By now Robert knew what to expect.

“Red again: $1 + 3 + 1 = 5$. Then yellow: $3 + 4 + 1 = 8$. And blue: $1 + 6 + 5 + 1 = 13$.”

“Tell me, what’s going on here with this 1, 1, 2, 3, 8, 13 . . . ?”



“We’re back to Bonacci and the rabbit numbers.”

“See how much we’ve packed into our triangle? And we could go on for days. But I have a feeling you’ve had enough.”

“More than enough, actually.”

“All right then,” he said, and with a clap of the hands he turned off the colored cubes. “A pity, though, because you know what I can do with just one more clap? I can light up the even numbers and leave the odd numbers dark. Are you game?”

“If you insist.”

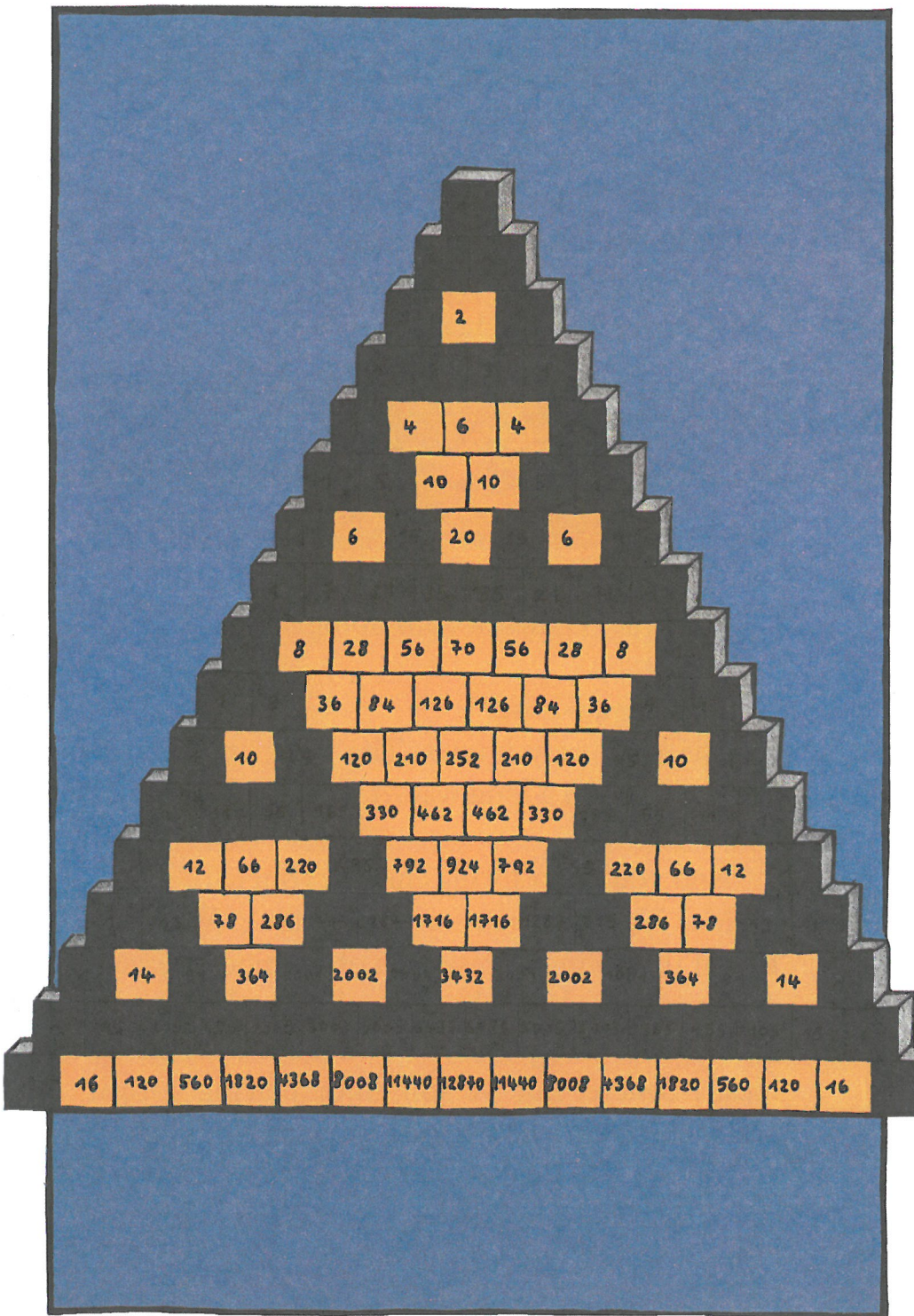
But Robert was amazed at what he saw.

“Hey, that’s wild! Triangle after triangle within the triangle! Except they’re upside down.”

“And come in small, medium, and large. The small looks like a single cube, but it’s actually a triangle; the medium consists of six cubes; the large of twenty-eight. Triangle numbers all.

“What do you think will happen if we turn off the even numbers, the numbers that can be divided by two, and light up the numbers that can be divided by three or five? All it takes is a clap of the hands. Which would you like to see? Shall we try five?”

“Yes,” said Robert. “All numbers divisible by five.”



When the number devil clapped his hands, the orange lights went out and green lights came on.

“Never in my wildest dreams would I have expected triangles again,” said Robert. “The same, but different. Pure witchcraft!”

“Yes, my boy. I often wonder where mathematics stops and witchcraft begins.”



“Fantastic! Is this all your doing?”

“No.”

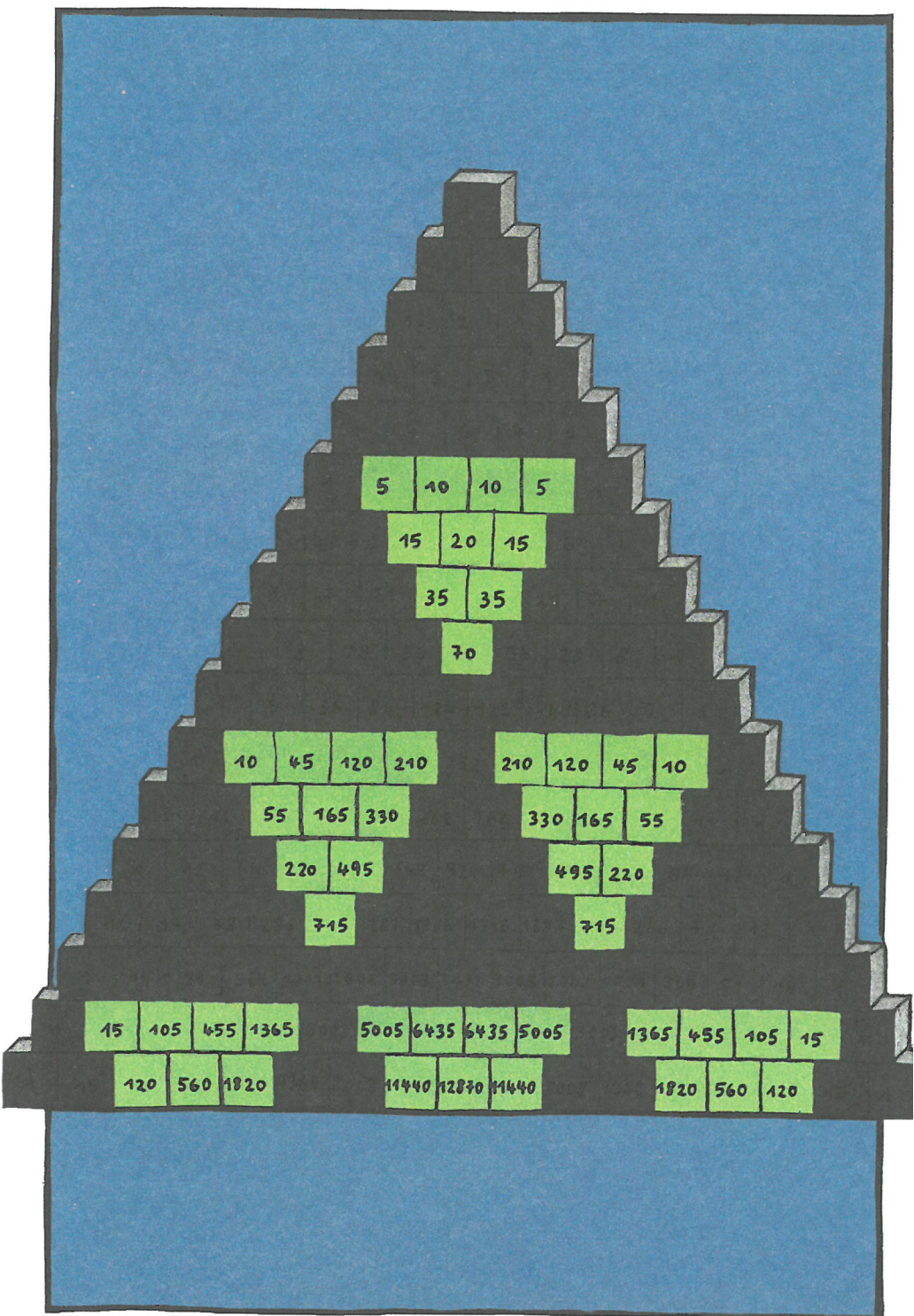
“Well then, whose?”

“The devil only knows. The great number triangle goes back a long way. It’s much older than I am.”

“And you’re no spring chicken yourself.”

“Me? How can you say such a thing! Why, I’m one of the youngest residents of Number Heaven. And our triangle is at least two thousand years old. It was a Chinese gentleman who came up with the idea, I believe. But we still enjoy playing with it and making it do new tricks.”

Nothing you do ever seems to have an end to it, thought Robert, not daring to say it out loud.



5 10 10 5

15 20 15

35 35

70

10 45 120 210

210 120 45 10

55 165 330

330 165 55

220 495

495 220

715

715

15 105 455 1365

5005 6435 6435 5005

1365 455 105 15

120 560 1820

11440 12870 11440

1820 560 120

But the number devil must have read his thoughts, because he said, “Yes, mathematics is an endless story. Keep digging and you keep coming up with new things.”

“You mean you can’t stop?” Robert asked.

“I can’t,” whispered the number devil, “but you . . .” And as he spoke, the green cubes grew paler and paler and he grew thinner and thinner until they went off altogether and he was only a shadow of his former self. Before long, Robert had forgotten everything: bright cubes, big triangle, Bonacci numbers, and even his friend the number devil.

“You’re looking pale this morning,” his mother said when he awoke after a long, long sleep. “Are you having nightmares again?”

“Not in the least.”

“Well, I’m worried about you.”

“Don’t be, Mother. You know the saying: The Devil is never so grim as he is painted.”



Are any of you curious about what kind of pattern we get when we light up all the numbers that can be divided by four? You don't need to be a number devil to figure it out. Just copy the triangle on the next page, and take a colored pencil and color in all the numbers that occur in the fours column of the multiplication table. With numbers above forty-eight use your calculator: enter the number, then \div , 4, and $=$, and see whether it comes out even.



