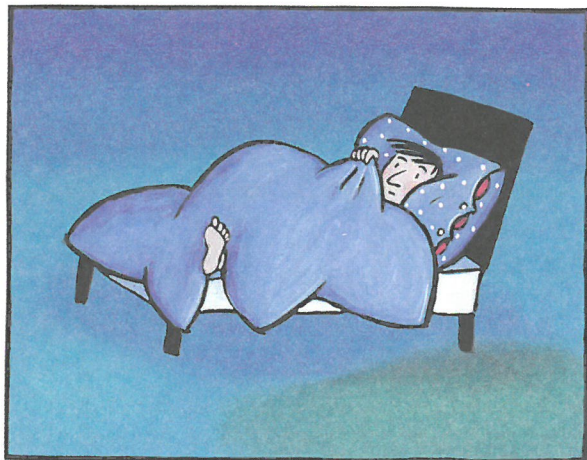


The Eighth Night





Robert was up at the blackboard. His two best friends in the class—Al, the one he played football with, and Betsy, the one with the pigtails—were sitting in front, as usual. And, as usual, they were having an argument.

Just what I needed, thought Robert. A dream about school!

At that very moment the door flew open and in came not Mr. Bockel but—the number devil.

“Good morning,” he said. “Arguing again, I see.”

“Betsy’s sitting in my place,” said Al.

“Then switch places.”

“She won’t budge.”

“Put an *A* for Al and a *B* for Betsy on the board, Robert,” said the number devil.

Why not, Robert thought, if it makes him happy.

AB

“Now, Betsy,” said the number devil, “I want you to change places with Al.”

And for some strange reason Betsy didn't make a scene. She stood up and changed places with Al.

BA

wrote Robert on the board.

At that very moment the door flew open again and in came Charlie, late as usual. Charlie sat down next to Betsy.

CBA

wrote Robert.

But Betsy didn't like that. “If I'm going to sit on the left, I'm going to sit all the way on the left.”

“Heavens to Betsy!” Charlie cried.

And the two changed places:

BCA

Then Al was upset. “I want to sit next to Betsy!” he said, so easygoing Charlie gave Al his place without a murmur:

BAC

If this keeps up, Robert said to himself, we can forget about the class.

And it did keep up, because Al decided *he* wanted to sit on the left.

“Which means we all have to get up,” said Betsy. “I don’t know why, but . . . Come on, Charlie.”

Here is how it looked when they’d all settled down again:

A B C

Not that it lasted, of course.

“I won’t sit next to Charlie for another minute,” said Betsy before long. She was a real pain, that Betsy. And since Betsy had to have her way, the boys had to get up and move. Robert wrote:

C A B

“Enough is enough,” he said. “That’s it.”

“Is it, now?” the number devil responded. “The three of them haven’t exhausted the possibilities open to them. Supposing Al were to sit on the left, Charlie in the middle, and Betsy on the right?”

“Not on your life!” Betsy cried.

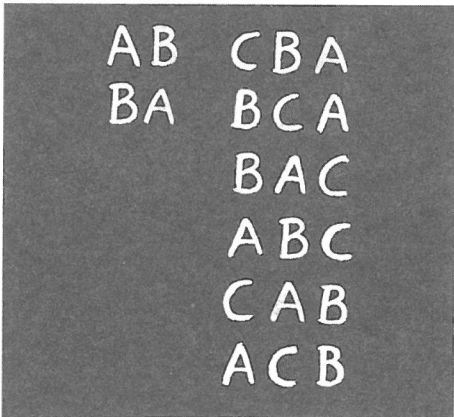
“Don’t make such a fuss, Betsy!”

The three of them pulled themselves up again and repositioned themselves:

A C B

“Hey, Robert! Notice anything? I don’t think those three will come up with anything else.”

Robert looked at the board:



AB CBA
BA BCA
BAC
ABC
CAB
ACB

“We seem to have tried all the possibilities,” he said.

“That we have,” said the number devil. “But there are more than the four of you in this class. Quite a few more, I’m afraid.”

At that very moment the door flew open and in ran Doris, out of breath.

“Hey, what’s going on here? Where’s Mr. Bockel? And who are you?”

“I’m substituting for Mr. Bockel. He’s taking the day off. He said he needed a rest from the chaos that reigns in this class.”

“I can see why,” said Doris. “Look at them. They’re all in the wrong seats. Since when do you sit *there*, Charlie? That’s my desk!”

“What order do you suggest, Doris?” the number devil asked.

“Why not go by the alphabet?” she said. “A for Al, B for Betsy, C for Charlie, and so on. That’s the simplest way out.”

“As you like. Let’s give it a try.”

So Robert wrote the following on the board:



A B C D

But the others weren’t the least bit happy with Doris’s suggestion, and all hell broke loose in the classroom. Betsy was the worst, biting and scratching when one of the others refused to give way, but they all pushed and shoved. Then, little by little, the crazy game of musical chairs they were playing began to seem like fun, and they switched places so fast that Robert had trouble keeping up with them. In the end, though, he did manage to get all their seating combinations on the board:



ABCD	BACD	CABD	DABC
ABDC	BADC	CADB	DACB
ACBD	BCAD	CBAD	DBAC
ACDB	BCDA	CBDA	DBCA
ADBC	BDAC	CDAB	DCAB
ADCB	BDCA	CDBA	DCBA

It's a good thing there are lots of kids absent today, thought Robert, or it would go on forever.

And at that very moment the door flew open and in came Enrique, Felice, Gary, Hugh, Iris, Jamil, and Karen.

"No! No!" Robert cried. "No! Please! Don't sit down or I'll go crazy!"

"All right," said the number devil. "That's it for today. You can all go home."

"Me too?" Robert asked.

"No, I need you to stay a while."

While his classmates ran out into the schoolyard, Robert looked over the numbers on the board.

"Well," said the number devil. "What do you make of them?"

"I'm not sure," said Robert. "All I know is that the number of ways they can sit increases awfully fast. As long as there were only two kids, things



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were simple: two kids, two possibilities. But with three kids there were six possibilities. And with four—wait a minute—twenty-four.”

“What if there’s only one?”

“What a thing to ask! Only one possibility.”

“Let’s try multiplying,” said the number devil.

Children:	Possibilities:
1	1
2	$1 \times 2 = 2$
3	$1 \times 2 \times 3 = 6$
4	$1 \times 2 \times 3 \times 4 = 24$
5	$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$

“I see,” said Robert. “Interesting.”

“The more of your classmates join in the game, the more inconvenient it is to write it out like that. There’s a shorter way, though. You take the number and put an exclamation mark after it. Like this:

$$4! = 24$$

Factorial

And you read it: four vroom!”

“What do you think would have happened if you hadn’t sent Enrique, Felice, Gary, Hugh, Iris, Jamil, and Karen home?”

“Utter confusion. Pandemonium,” replied the number devil. “I can just see them pushing and shouting, trying out each and every combination. It would have taken ages. Together with Al, Betsy, and Charlie there would have been eleven of them. That means eleven vroom! possibilities. Can you guess how many that is?”

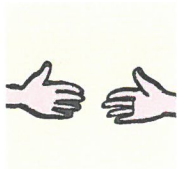
“I know I can’t do it in my head, but I always bring my calculator to school—I have to hide it, of course: Mr. Bockel can’t stand the sight of calculators—so I’ll have the answer for you in a jiffy.

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 =$$

“Eleven vroom!” he said excitedly, “is precisely 39,916,800. Wow! Nearly 40 million!”

“So if we’d gone through all the combinations, we’d still be here eighty years from now. Your classmates would be in wheelchairs and we’d have to hire eleven nurses to do the pushing. See how useful a bit of mathematics can be? Which reminds me . . . Have a look out of the window and tell me if your classmates are still there.”

“Oh, I’m sure they’ve all gone their separate ways.”



“I assume you shake hands when you say good-bye.”

“Shake hands! We mumble, ‘See you’—if you’re lucky.”

“A pity,” said the number devil, “because I wonder how long it would take for each of them to shake hands with each of the others.”

“You know perfectly well there’d be an untold number of handshakes. Eleven vroom! of them, I suppose, since there are eleven of them.”

“Wrong!” said the number devil.

“Wait a minute,” said Robert. “I see. If there were two of them, they’d need only one handshake. If there were three . . .”

“Try putting it on the board.”

This is what Robert wrote:

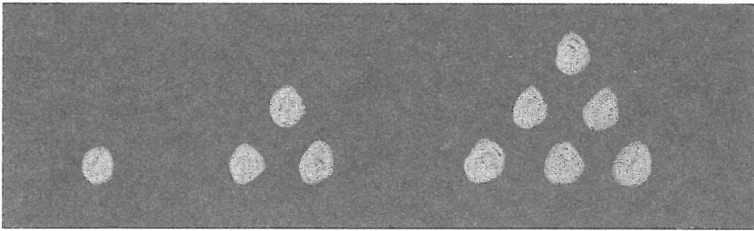
Pe	Handshake
1	0 > +1
2	1 > +2
3	3 > +3
4	6 > +4
5	10 > +5
6	15

People:	Handshakes:
A	—
AB	AB
ABC	AB AC BC
ABCD	AB AC AD BC BD CD

“Two people—one handshake. Three people—three handshakes. Four people—six handshakes. Five people—ten.”

“One, three, six, ten . . . Look familiar?”

Robert couldn't remember, so the number devil made a few big dots on the board:



“Coconuts!” Robert shouted. “Triangle numbers!”

“And how do they go?”

Robert wrote on the board:

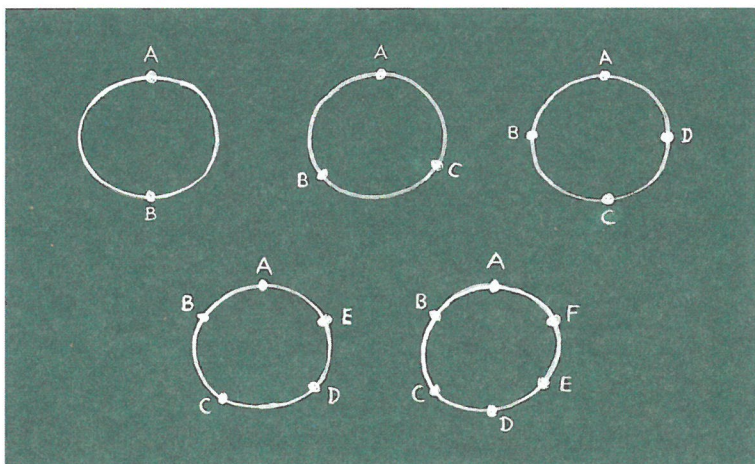
$$\begin{array}{r} 1 + 2 = 3 \\ 3 + 3 = 6 \\ 6 + 4 = 10 \\ 10 + 5 = 15 \\ 15 + 6 = 21 \\ 21 + 7 = 28 \\ 28 + 8 = 36 \\ 36 + 9 = 45 \\ 45 + 10 = \end{array}$$

“So you'd need exactly fifty-five handshakes.”

“That wouldn't be so bad,” said Robert.

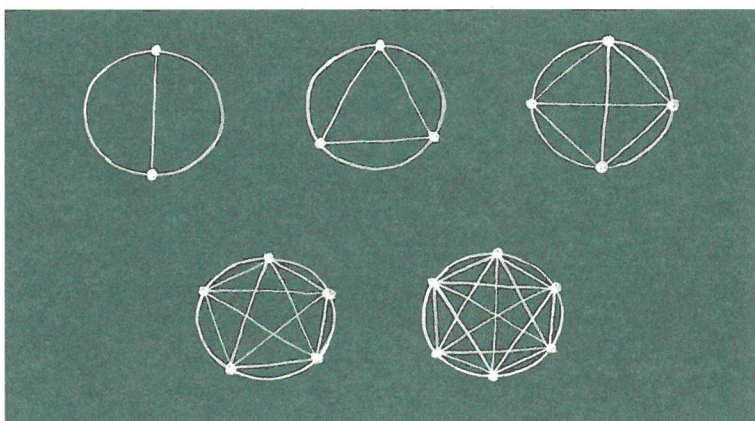
“And here's what you do to get around all that

arithmetic. You draw a few circles on the board:



The letters stand for your friends: *A* for Al, *B* for Betsy, *C* for Charlie, and so on.

“Then you join the letters with lines:



“Pretty, isn’t it? And since each line represents a handshake, all you have to do is count them.”

“One, three, six, ten, fifteen . . . As usual,” said Robert. “There’s only one thing I don’t understand. How is it that everything you do works?”

“That’s the devilish thing about numbers: everything works. Well, almost everything. Because the prima-donna numbers—remember them?—they have their problems. And you’ve got to keep your eyes peeled for them or you’ll fall flat on your face. Which is why so many people hate numbers. I can’t stand slobs, and they can’t stand numbers. By the way, go to the window and you’ll see a schoolyard that looks like a pigsty.”

Robert had to admit it. The schoolyard was strewn with soda cans, newspapers, and sandwich wrappings.

“If three of you pick up some brooms, you can sweep it clean in half an hour.”

“Which three have you got in mind?” Robert asked.

“Al, Betsy, and Charlie, say. Or Doris, Enrique, and Felice. And you’ve got Gary, Hugh, Iris, Jamil, and Karen waiting in the wings.”

“So *which* three doesn’t matter.”

“Right.”

“Then we can combine them any old way,” said Robert.

“Right again. But supposing they’re not all available. Supposing Doris, Enrique, and Felice aren’t available, so we have only three: Al, Betsy, and Charlie.”

“Then they’ll have to do it.”

“Good. Put that on the board.”

And Robert wrote:

ABC

“Now, if Doris runs in late as usual, what do we do? What are the possibilities?”

Robert thought a moment and wrote the following:

ABC ABD ACD BCD

“These four,” he said.

“Now Enrique turns up. Why shouldn’t he be included? That makes five candidates. See what you can do with five.”

But Robert refused. He was getting a little nervous.

“You tell me,” he said.

“All right. With three people we can have only one group of three; with four we can have four. And with five we can have ten. Here, let me put it on the board:

People:

Groups:

3	ABC									
4	ABC	ABD		ACD			BCD			
5	ABC	ABD	ABE	ACD	ACE	ADE	BCD	BCE	BDE	CDE

“There’s something special I want you to notice. As you can see, I’ve put the groups in alphabetical order. How many groups begin with Al? Ten. How many with Betsy? Four. And with Charlie only one. The same numbers keep coming up:

1, 4, 10 ...

Can you guess how it goes on from there? I mean, if we add a few more names: Felice, Gary, Hugh, and so forth. How many groups would we have then?”

“Beats me,” said Robert.

“You remember how we cracked the handshake

problem? When everybody shakes hands with everybody else?”

“That was a breeze. We used the triangle numbers:

1, 3, 6, 10, 15, 21...

But they won't help with our broom brigades, which work three to a group.”

“Now what if you add the first two triangular numbers together?”

“That makes four.”

“And the next one.”

“That makes ten.”

“And the next.”

“ $10 + 10 = 20$.”

“Go on.”

“You mean keep going until I get to the eleventh? You can't be serious.”

“Don't worry. You can get there without arithmetic. You can get there without guessing, even without *ABCDEFGHIJK*.”

“How?”

“With our good old number triangle,” said the number devil.

“You mean you're going to put one on the board?”

“Heavens no! Not when I have my walking stick handy.”

No sooner did he tap the stick on the board than there it was, in all its glory. In glorious color too.

“Couldn’t be easier,” he said. “For the handshakes you count from top to bottom using the green cubes: one handshake for two people, three for three people, and fifty-five for eleven people.

“For our broom-brigade trio you use the red cubes, again from top to bottom. We start out with three people and one possibility. When we have four people to choose from, we have four combinations. With five people we have ten. How many would we have if all eleven of your classmates show up?”

“165,” Robert answered. “You’re right. It *is* easy. This number triangle is nearly as good as a computer. But tell me, what are the orange cubes for?”

“Oh, them,” said the number devil. “Well, as you may have noticed, we number devils aren’t easily satisfied; we tend to go overboard. So just in case three people couldn’t handle the clean-up and you needed a fourth, I wanted you to know how many possibilities you’d have. How many will there be if, say, eight people apply for a broom-brigade quartet?”

“Seventy,” said Robert, having no trouble finding the answer in the number triangle.

“Correct,” said the number devil. “By now you can guess what the blue cubes are for.”



“The broom-brigade octet,” said Robert. “If I have eight volunteers, I have only one possibility. But with ten I have forty-five. And so on.”

“You get the picture.”

“What do you think the schoolyard looks like now?” Robert said, looking out of the window. It was cleaner than he had ever seen it. “I wonder which three did it?”

“Well, you weren’t among them, my dear Robert.”

“How do you expect me to sweep the schoolyard when you keep throwing numbers and cubes at me all night long?”

“Maybe you need a little rest from me.”

“What do you mean? Aren’t you coming back?”

“I thought I could use a little holiday,” said the number devil. “You can always talk numbers with Mr. Bockel.”

It wasn’t the greatest prospect in the world, but he had no choice. And he had to go back to school the next day anyway.

When he walked into the classroom, he saw Al and Betsy in their normal places. Neither seemed keen on switching.

“Here comes the wizard!” Charlie called out.

“Robert does problems in his sleep!” Betsy teased.



“Do you think it helps?” asked Doris.

“Not very much,” said Jamil. “Mr. Bockel can’t stand him.”

“Well, the feeling’s mutual,” Robert replied.

Robert stole a glance at the schoolyard.

As usual, he thought. A regular dump! So much for his dream. But the numbers remained. He could count on the numbers.

At that moment the door flew open and in walked the inevitable Mr. Bockel with a briefcase chock-full of pretzels.

