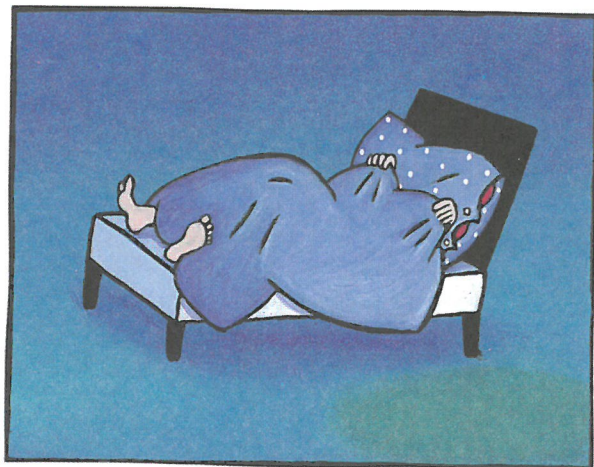


The Ninth Night





Robert dreamed he was dreaming. He was used to it by now. Whenever he had a dream about something unpleasant—the one about being stranded on a slippery rock in the middle of a raging stream, for instance—he would think to himself, Horrible as it is, it’s all a dream.

But one day he caught the flu, and, lying in bed all day with a temperature, he found the trick didn’t work. Besides, fever dreams were the worst. The last time he’d been sick in bed he’d had a dream about a volcano erupting. Fire-spewing mountains had flung him into the air, and he was about to descend slowly—slowly, how curious—into the maw of the volcano. It gave him the creeps just to think of it. So he tried to stay awake, even though his mother kept telling him, “The best thing to do is to sleep it off. Don’t read so much. It’s unhealthy!”



After the twelfth comic book, however, his eyes were so heavy that they closed by themselves, and what he dreamed was as strange as strange could be.

He dreamed he was in bed with the flu and the number devil was sitting next to him. There was a glass of water on his bedside table and he thought, I'm hot. I have a temperature. I don't think I'm asleep.

"What about me?" asked the number devil. "Are you dreaming me or am I really here?"

"I'm not sure," said Robert.

"What difference does it make?" said the number devil. "I'm just making a sick call. And since you're ill and must stay in bed and can't climb trees in the desert or count rabbits in the country, I thought we'd spend a quiet evening at home. I've brought along a few numbers to take your mind off things. They're perfectly harmless, I assure you."

"That's what you always say."

Just then there was a knock on the door.

"Come in!" the number devil called out.

And in they marched. They reminded him of racing cyclists or marathon runners, because they sported their numbers on white T-shirts. They came in such quantities that before he knew it,

Robert's room was packed. At first he was amazed to see so many squeeze into so small a space, but then he realized that as more and more crowded in, the door moved farther and farther away, until it stood at the end of a long, narrow corridor and he could hardly make it out.

For a while the numbers just stood there laughing and chattering away. Then the number devil shouted in his best army-sergeant voice, "Attention! First row, fall in!" and they immediately lined up, backs to the wall, one at the head, the others following in numerical order.

"Where's zero?" Robert asked.

"Zero, front and center!" the number devil roared.

Zero had hidden under the bed and crawled out, terribly embarrassed.

"I thought I'd not be needed. I'm not myself today. I must be coming down with the flu. I'm afraid I'll have to ask for sick leave."

"Dismissed!" the number devil shouted, and zero crept back under Robert's bed.

"That zero! Always making problems, wanting something special. But the others—I hope you appreciate how well they follow orders."

He seemed tremendously pleased with a line of perfectly ordinary numbers:



1	2	3	4	5	6	7	8	9	10	11	12	13
---	---	---	---	---	---	---	---	---	----	----	----	----

“Second row, fall in!” he shouted, and immediately a new contingent of numbers stormed in and found their places with a great clatter and shuffle:

1	3	5	7	9	11	13	15	17	19	21	23	25
---	---	---	---	---	----	----	----	----	----	----	----	----

They stood directly in front of the others, making the room look even more like an interminable tunnel. They were all decked out in identical red T-shirts.

“I see,” said Robert. “The odd numbers.”

“Right. Now I want you to guess how many of them there are compared with their white-shirted comrades along the wall.”

“That’s obvious,” said Robert. “Every other number is odd, so there are half as many reds as there are whites.”

“What you’re saying is that there are twice as many ordinary numbers as odd.”

“Right.”

The number devil laughed, but it wasn’t a nice laugh. Robert thought it sounded sarcastic.

“Sorry to disappoint you, my boy, but as you see, there are exactly the same number of one as of the other.”

“Ridiculous!” Robert cried. “*All* can’t be the same as *half*.”

“Watch carefully and I’ll show you what I mean.”

He turned to the numbers and roared, “First and second rows, shake hands!”

“You don’t need to scream at them, do you?” Robert said angrily. “This isn’t an army barracks. Try being a little more polite.”

But his protest went unheeded, because by then they had formed pairs like tin soldiers and each white was shaking hands with a red:

1	2	3	4	5	6	7	8	9	10	11	12	13	...
1	3	5	7	9	11	13	15	17	19	21	23	25	...

“See? Each ordinary number from one on has its own odd number from one on. Can you show me a single red without a partner? So there is an infinite quantity of ordinary numbers and an infinite quantity of odd numbers. Infinite, understand?”

Robert thought for a while.

“So if I divide an infinite quantity in half I get two infinite quantities. But then the whole is the same size as the half.”

“Correct,” said the number devil. “And not only that.” He pulled a whistle out of his pocket and gave a toot. All at once a new column of numbers—this one in green T-shirts—appeared out of the depths of the endless room, jiggling and joggling until the number devil commanded, “Third row, fall in!”

In a flash the greens formed a neat line in front of their red and white comrades:

2	3	5	7	11	13	17	19	23	29	31	37	41
---	---	---	---	----	----	----	----	----	----	----	----	----

“Prima donnas,” Robert concluded from the numbers on their T-shirts.

The number devil merely nodded. Then he gave another toot on his whistle and another and another and another. All hell broke loose. A nightmare! Who would have thought that so many numbers could fit in a single room, even if it had by now grown as long as the path a rocket takes to the moon. There was no air left. Robert’s head felt like a glaring lightbulb.

“Stop! Stop! I can’t take any more of this!”



"Come in!" the number devil called out, and in marched the numbers in such quantities that before Robert knew it, his room was packed

“Your flu must be getting to you,” said the number devil. “I’m sure you’ll feel better tomorrow.”

Then he turned to the numbers and shouted, “Now hear this! Rows four, five, six, and seven, fall in! On the double!”

Robert forced his drooping eyes open and saw seven kinds of numbers in white, red, green, blue, orange, black, and pink T-shirts standing one behind the other in neat but endless rows:

<i>all</i>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
<i>odd</i>	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29
<i>prime</i>	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47
<i>Fibonacci</i>	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610
<i>Triangular #'s</i>	1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
2^x	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	
$X!$	1	2	6	24	120	720	5040	40320	362880	3628800	36288000	39916800			

The numbers on the pink T-shirts were soon so long that they barely fit, and it was all Robert could do to read them.

“They get large so quickly! I’ll never keep up.”

“Vroom!” said the number devil. “The numbers with the exclamation mark:

$$3! = 1 \times 2 \times 3$$

$$4! = 1 \times 2 \times 3 \times 4$$

And so on. Hard to keep up with them, isn't it? But what about the others? Do you recognize them?”

“Let's see. The reds are odd, the greens prima donnas, the blues—I don't know, but they look familiar.”

“Think rabbits.”

“Oh, yes. The Bonaccis. Which would make the orange numbers triangle numbers.”

“Not bad. Flu or no flu, you're making progress.”

“The blacks are obviously hopping numbers: 2^2 , 2^3 , 2^4 , and so on.”

“And there is an equal quantity of each color,” said the number devil.

“An infinite quantity,” said Robert with a sigh. “Awful, isn't it. A real mob scene.”

“Rows one through seven, dismissed!” the number devil roared.

In a flurry of scraping, pushing, puffing, and trampling, the numbers left the room. They were replaced by an exquisite stillness. Robert's room was as small and bare as before.

“All I need now is an aspirin and a glass of water.”

“Plus a good rest, and you’ll be back on your feet tomorrow,” said the number devil, gently tucking Robert in.

“But do you think you can keep your eyes open long enough to take care of what we have left?”

“Left of what?”

“The thing is,” he said, waving his stick again “we booted the numbers out because they made such a mess of your room. But we still have series to deal with.”

“Series? What are series?”

“Well, you don’t think numbers just stand there like tin soldiers, do you? What happens when they are combined—that is, when they’re added together?”

“I don’t know what you’re talking about,” Robert moaned.

But by then the number devil had drawn the first series on the ceiling with his stick.

“I thought you said I needed rest.”

“This won’t take much out of you. All you have to do is read what it says.”

“Fractions!” Robert moaned again. “Yuck!”

“What do you mean? What could be simpler than fractions? Look at these!”



When Robert looked up at the ceiling again, the numbers were gone and had been replaced by a long line.

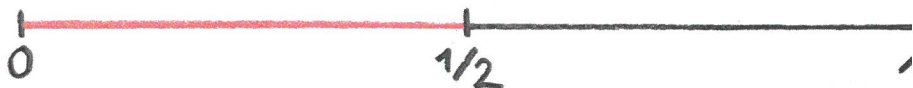
$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} \dots =$$

“One-half,” Robert read, “plus one-quarter, plus one-eighth, plus one-sixteenth, plus one-thirty-second, and so on. Ones on top and hopping twos on the bottom. The same as the black T-shirts: two, four, eight, sixteen, thirty-two . . . And I’m sure I know what comes next.”

“Yes, but what comes from adding them all together?”

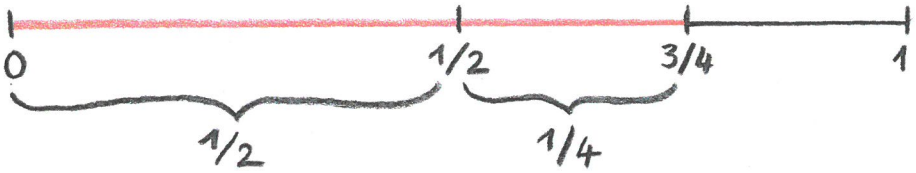
“I have no idea, though if the series never ends then what comes of it must be never-ending too. On the other hand, one-quarter is less than one-half, one-eighth is less than one-fourth, and so on. So the numbers I add on will get smaller and smaller.”

When Robert looked up at the ceiling again, the numbers were gone and had been replaced by a long line:

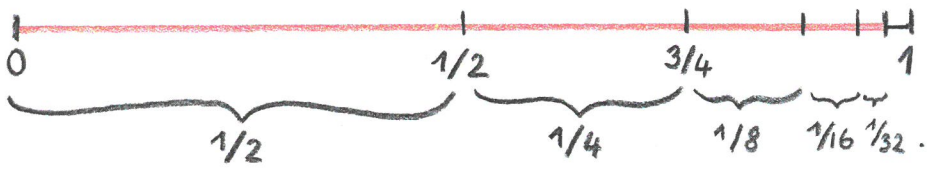


“I think I get it,” he said, after staring up at it for a while. “You start with one-half, then add on half of one-half, in other words, one-quarter.”

As he spoke, the numbers appeared in black and white on the ceiling:



“And you just keep going, adding on half of the previous number. Half of one-quarter is one-eighth, half of one-eighth is one-sixteenth, and so on. The pieces will get smaller and smaller, so small that they’ll soon be invisible, like the pieces of chewing gum you divided up that first night.”



You can keep going till you’re blue in the face and you’ll never reach the one. Almost, but never quite.”

“Well, I want you to keep going.”

“And I don’t want to. I’m in bed with the flu, remember?”

“That’s just the point. *You* may get tired, but numbers don’t,” the number devil said. “They can go on and on forever.”

Suddenly the line on the ceiling was replaced by the following:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} \dots = 1$$

“Well done!” cried the number devil. “Excellent! Keep going.”

“But I’m tired. I need to sleep.”

“Sleep?” said the number devil. “You *are* sleeping. You’re dreaming of me, aren’t you? And you can only dream if you’re sleeping.”

There was nothing Robert could say to that, though he felt his brain was turning into jelly.

“All right, I’ll go along with *one more* of your crazy ideas, but then I’ve got to rest.”

The number devil raised his stick, snapped his fingers, and a whole new series appeared on the ceiling:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots =$$

“It’s just like the last one,” Robert said. “I can go on adding till the cows come home, but since each number is smaller than the one before it, they’ll never add up to one.”

“Is that what you think? Then let’s look a bit closer. At the first two numbers, for instance.”

Now only the first two numbers of the series were left on the ceiling:

$$\frac{1}{2} + \frac{1}{3}$$

“What’s the answer?”

“I don’t know,” Robert muttered.

“Don’t act stupid now. Which is more? One-half or one-third?”

“One-half, of course!” Robert said, annoyed. “What do you take me for?”

“Now, now. Just tell me this: Which is more? One-third or one-fourth?”

“One-third, of course.”

“So we have two fractions, both of which are more than one-fourth. And what do two-fourths make?”

“What a dumb question! Two-fourths make a half.”

“Good.”



$$\frac{1}{2} + \frac{1}{3} \quad \text{is therefore} \quad \frac{1}{4} + \frac{1}{4}$$

more than

And if we take the next four terms of the series and add them together, they too come out to be more than one-half. Look:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

“That’s too complicated for me,” Robert grumbled.

“Nonsense!” cried the number devil. “Which is more? One-fourth or one-eighth?”

“One-fourth.”

“Which is more, one-fifth or one-eighth?”

“One-fifth.”

“Right. And the same holds for one-sixth and one-seventh. See these following fractions?”

$$\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$$

They are all more than one-eighth. And what do four-eighths make?”

Robert did not even want to answer, but he finally said, “Four-eighths make exactly one-half.”

“Excellent. So now we have

$$\underbrace{\frac{1}{2} + \frac{1}{3}} + \underbrace{\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}} + \underbrace{\frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \dots + \frac{1}{15} + \dots}$$

more
than $\frac{1}{2}$

more
than $\frac{1}{2}$

more
than $\frac{1}{2}$

And so on. Till the cows come home. You’ll notice that if we add the first six terms of the series together they come to more than one. And we can go on like this as long as we like.”

“No, no!” Robert cried. “Please!”

“But *if* we went on—don’t worry, we won’t—where would it take us?”

“To infinity, I suppose,” said Robert. “How devilish of you.”

“Except that it would take forever. Even if we worked at lightning speed, we wouldn’t reach the first thousands till, say, the end of the world. That’s how slowly the series increases.”

“Then let’s leave well enough alone.”

“Yes, let’s leave well enough alone.”

And with that the writing on the ceiling began to fade, the number devil grew thinner and thinner, and time moved on.

Robert did not wake up until the sun was tickling his nose.

“Thank God the fever is gone,” his mother said, putting her hand on his forehead.

By then he had forgotten how easy and how slow it can be to slide from one to infinity.

